The Informational Content of Unemployment: Equilibrium Forces and Dynamics

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Abstract

In the standard analysis of employment dynamics, workers reach unemployment after being fired. Firing standards rise during recessions, suggesting that the unemployment pool quality rises as well. I show that this is incorrect—a proper analysis of unemployment must also incorporate job leavers. Firings increase relative to quits during recessions, and I present empirical evidence that compositional shifts of this sort result in lower quality workers entering unemployment. I then develop a model of labor market equilibrium in which these compositional shifts arise endogenously, and I study the consequences for employment dynamics. The quality of the unemployment pool declines during recessions, and firms limit hiring in response. For hiring to return, unemployment pool quality must recover via inflows of higher quality job leavers. In a significant recession, this recovery may be very slow—aggregate demand may return to pre-recession levels before unemployment pool quality does. This offers an explanation for jobless recoveries. The model also reconciles other observed empirical patterns, including countercyclical average labor productivity, a negative relationship between hiring probabilities and unemployment duration, and a convergence of hiring probabilities between the short- and long-term unemployed following increases in firings and decreases in quits.

Several years after the "official" end of this past recession, over 8% of the U.S. labor force remains unemployed, and 43% of these unemployed have been jobless for over six months (BLS - CPS Labor Force Statistics). Within the unemployment pool, the long-term unemployed workers (LTU) are least likely to find jobs, and policy-makers worry that the current mass of LTU could be trapped in this state. As such, LTU must be a primary target of any policy to reduce unemployment, but the effectiveness of such a policy depends on why these workers struggle to find jobs. In the existing literature, the lower reemployment probabilities of LTU are attributed to human capital depreciation during unemployment (Pissarides, 1992; Ljungqvist and Sargent, 2008; Möller, 1990), negative sorting induced by selective hiring from the unemployment pool (Lockwood, 1991), and even employer bias against LTU caused by the assumed presence of the previous two mechanisms (Jackman and Layard, 1991).

In this paper, I consider a new factor contributing to the current job-finding struggles of LTU: changes over time in the quality of workers entering the unemployment pool. Specifically, more job losers and fewer job leavers enter unemployment during recessions, and as a result, less productive people become unemployed at such times. The analysis to follow develops this idea, offering two main contributions: First, I provide empirical evidence for the aforementioned changes in quality. Second, I present an equilibrium model of the labor market in which these changes arise endogenously during recessions, and I use this model to analyze the short and long-term consequences for employment dynamics.

In the empirical section, I show that more job losers and fewer job leavers enter unemployment during recessions, and I provide evidence that job leavers are of higher quality than job losers. These patterns suggest that the unemployment pool declines in quality during recessions. I further support this mechanism with direct evidence that shifts from quits toward firings lower the quality of the unemployment pool. This evidence is based on the following intuition:

Because firms hire selectively, better quality workers are more likely to be hired. This implies that worker reemployment probabilities correlate with unobserved quality. Further, if we divide the unemployed into into short-term unemployed workers (STU) and LTU, then workers should be counted among STU when they first enter the unemployment pool. Thus, if more job losers and fewer job leavers enter the unemployment pool, then the quality of those entering unemployment should decline, and the reemployment probabilities of STU should decline relative to those of LTU. I confirm this empirically using CPS micro-data, and the result persists regardless of whether the numbers of job losers and leavers come from the employer-reported Job Openings and Labor Turnover Survey (JOLTS) or from the employee-reported CPS monthly sample.

This empirical approach is particularly informative for two reasons: first, the main explanations for long-term unemployment found in the literature (human capital depreciation, selection among those remaining unemployed, etc.) cannot—by themselves—generate this empirical pat-

tern. While these alternative mechanisms are important for understanding unemployment, this analysis identifies a role for compositional change that is independent of these existing explanations. Second, the proportions of firings and quits entering unemployment are linked directly to hiring outcomes, which are a primary welfare objective of policy makers. Thus, from a welfare perspective, any alternative explanation for these findings should have similar implications regarding unemployment.

With this empirical motivation, I develop a model of labor market equilibrium in which these compositional changes arise endogenously during recessions, and I analyze the implications of these changes for employment dynamics. In the model, firms hire selectively from a pool of heterogeneous, unemployed workers. This selective hiring is imperfect, and firms then learn privately during employment about worker productivity. Thus, we can parametrize both how much information about worker quality firms can obtain before hiring and how quickly firms obtain this information after hiring.

Workers can reach the unemployment pool either by voluntarily quitting or by being targeted for firing. Firms fire workers due to low beliefs about productivity, so these workers are negatively selected, but this selection does not apply to job leavers. In equilibrium, the quality of those entering unemployment reflects a balance between low quality job losers and better quality job leavers.

Using this framework, I study the labor market dynamics induced by a recession, in which workers become less productive relative to their costs of employment. The analysis offers three main insights:

(1) Firms respond to this "shock" by raising standards for firing current workers and for employing new ones. Thus, consistent with the standard analysis in the literature (Nakamura, 2008; Kosovich, 2010; Lockwood, 1991), workers fired during recessions are of higher average quality than those fired under other economic conditions. However, a recession also throws off the preexisting balance between fires and quits—the flow of job losers overwhelms that of job leavers. As a result, a recession *decreases* the quality of the unemployment pool, reversing the standard conclusion.

(2) This drop in quality lowers each firm's expected value of hiring a new worker from the unemployment pool, so firms will limit hiring. For hiring to return, the unemployment pool quality must rebound through inflows of workers who have not been targeted for low productivity, such as job leavers. The unemployment pool's quality may take a long time to recover, and this may be further delayed if poor job-finding conditions motivate fewer workers to quit voluntarily. In fact, this lower quality unemployment pool may continue to suppress hiring even if there is a positive productivity shock and the economy otherwise

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recovers to pre-recession conditions. Thus, this may help explain the "jobless recoveries" that have followed recent recessions.[1]

(3) More generally, the above results highlight the importance of explicitly modeling economic dynamics in this context, as conclusions drawn from comparing the predictions of two static models may be misleading. In this case, while the unemployment pool in the new steady-state equilibrium associated with a recession is of higher average quality than the pool in the pre-recession steady-state, the unemployment pool during the transition to the recession is of lower quality than that found in either steady-state. The model developed in this paper allows us to address such issues and take dynamics seriously—it offers a tractable equilibrium framework that can be used to study the evolution of employment and/or wages under changing economic conditions.

The paper proceeds as follows: Section 1 overviews the literature on various related topics. Section 2 reviews the standard view that recessions improve the unemployment pool's quality and presents evidence that the reverse is true. The next several sections develop a model of labor market equilibrium—Section 3 sets forth the structure of the economy, and Section 4 characterizes its steady-state equilibrium. Section 5 develops the dynamics of employment in response to a negative productivity shock (a "recession"), highlighting the causes and implications of the unemployment pool's changing composition. This section also considers a transitory shock and demonstrates how a jobless recovery can follow. Section 6 briefly considers several theoretical extensions and their implications, and Section 7 concludes.

1. Related Literature

The contributions of this paper relate to several literatures:

Employer Learning in Equilibrium

Since Jovanovic's seminal 1979 paper, learning has been a relevant consideration in models of labor markets. The importance of formally modeling learning has been demonstrated in a number of settings (see Farber and Gibbons 1996, and Altonji and Pierret 2001 for examples); simplified, *ad hoc* representations of learning may fail to capture important implications of the information structure. More specifically, this paper contributes to a recent literature analyzing learning in dynamic equilibrium environments (Anderson and Smith, 2010; Eeckhout and Weng,

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[1]A related prediction of the model is that employees remaining with a firm after a recession are disproportionately more productive than those employed beforehand. This mitigates the productivity decline that accompanies a recession, and may provide an explanation for the recently-observed acyclicality/countercyclicality of average labor productivity (see Gali and van Rens, 2010).

2010). The models in both of these existing papers are designed to analyze assortative matching, and they are not well-suited to the settings I study. In particular, Anderson and Smith allow individual-specific reputations to persist across matches, while Eeckhout and Weng allow sorting to take place through wage offers. In contrast, I consider a environment in which firms cannot separate heterogeneous workers from an unemployment pool, and in which firms have only aggregate information about this pool.

From a theoretical perspective, the Poisson learning I use is in many ways more tractible than the Brownian motion-based learning used in other continuous-time models (such as Jovanovic 1979, or Eeckhout and Weng 2010). As such, the model I present allows more precise characterization of equilibrium outcomes than was possible in previous models; this framework may offer insights beyond those directly mentioned in this paper.

Additionally, empirical evidence suggests that the asymmetric learning I model is relevant in labor markets. The findings of Kahn (2009) and Pinkston (2009) support the presence of asymmetric learning during employment; in particular, Pinkston (2009) suggests that asymmetric learning is at least as important in this context as the public learning modeled by Anderson and Smith. This type of asymmetric learning warrants further investigation, and my analysis contributes to this cause.

Adverse Selection in Labor Markets

An assortment of research has studied aspects of adverse selection in labor markets.[2] Gibbons and Katz (1991) began an empirical literature investigating differences between workers who lost jobs due to plant closings and those who lost jobs at plants which remained open ("layoffs"). They find that layoffs experience longer unemployment spells and lower wages upon reemployment. This is evidence that layoffs are of lower quality, but two underlying causes could drive this result. (1) Firms may stigmatize these workers based specifically on information about why they lost jobs. As a result, there may be explicit statistical discrimination against these workers. (2) Suppose firms observe other worker traits—not including reason for unemployment—that correlate with quality. Then firms will hire selectively and offer reemployment wages based on these traits. If some of these firm-observed traits are unobservable to the econometrician, then higher quality groups (like non-layoffs) will have better reemployment outcomes empirically. In my analysis, both of these mechanisms impact employment dynamics similarly, so I do not

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[2]On the theoretical side, several papers have studied adverse selection between firms in more basic settings than the dynamic equilibrium structure of my model. Laing (1994) considers how adverse selection in turnover between firms affects contracts within the firm. Because firms release their lowest quality workers, outside firms infer that those remaining are better. As a result of this positive signal, remaining workers must be compensated, and this distorts the optimal contract. Waldman (1984) and Greenwald (1986) show that the negative signal sent by workers leaving jobs can inhibit turnover between firms.

distinguish between them.[3]

More closely related to my analysis are Lockwood (1991) and Boone and Watson (2007), who investigate employer incentives to screen prospective workers and how these incentives affect the equilibrium unemployment pool. Screening is costly to firms, and it is associated with labor market externalities—screening can worsen the unemployment pool from which other firms hire. As in my model, worker quality declines with unemployment duration due to selective hiring.[4]

Both of these models assume that frictions arise from a search/matching process. In both cases, these frictions contribute to unemployment. In contrast, my model has no search frictions— workers will be hired immediately whenever firms can obtain *ex ante* profits from doing so. Surprisingly, my model generates equilibrium unemployment in spite of this. I show that selective hiring worsens the unemployment pool, and that this is sufficient to limit employment by reducing the *ex ante* value of hiring.

Additionally, these models study the consequences of screening only in a steady-state equilibrium; extending them to analyze a dynamic environment would be problematic. Boone and Watson assume that firms immediately learn worker types after hiring, but that these firms must wait a fixed duration before firing these workers. As a result, firings would not increase during a recession, but hiring would decrease, so there could be less negative selection among the remaining unemployed. Thus, recessions could actually raise unemployment pool quality in this model. In Lockwood's model, recessions could raise unemployment pool quality for a much simpler reason—there is no voluntary quitting.

Human Capital Depreciation

Human capital depreciation is a standard mechanism used to explain the persistence of unemployment following recessions.[5] A seminal example of this is Pissarides (1992), whose mechanism relies on a "thin market externality." A temporary negative shock that reduces hiring will raise unemployment durations. Because workers skills deteriorate during unemployment, this shock lowers the future returns to firms of searching for unemployed workers to hire. As a

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[3] My main analysis holds independently of which of these mechanisms is responsible for this result. Further, my findings are completely unchanged as long as the second cause plays any role in the different outcomes between fired workers and quitting workers (as long as these differences do not result exclusively from the first cause).

[4] I do not explicitly model firm investment toward more precise screening. This has important consequences for steady-state outcomes—it can even result in multiple equilibria with different levels of screening and different qualities of the unemployment pool. In the context of employment dynamics and economic shocks, however, this would not substantively change my analysis, so I omit it for simplicity.

[5] Berger (2012) offers another explanation for this, though this explanation is unrelated to the unemployment pool (in fact, there is no unemployment in Berger's model). He argues that selective firing during recessions increases firm efficiency, so firms enter recoveries better able to meet growing demand without hiring additional workers.

result, fewer firms enter the market in the period after the shock, and this further increases the durations of unemployed workers. In turn, worker skills deteriorate further, and the temporary shock is amplified.

Thus, via human capital depreciation, a recession can generate a mass of LTU who remain jobless long after other aspects of the economy have recovered. This result is often used to explain the "jobless recoveries" that have followed recent recessions (Ljungqvist and Sargent, 2008).

In this mechanism, workers who have been unemployed for a given duration do not change in quality over the business cycle. Instead, the distribution of unemployment durations changes— LTU are less productive, and there are more LTU in the aftermath of a recession. Thus, this standard human capital analysis ignores the impact of changes in the sources of unemployment (and, in turn, in the productivity of the unemployed) over the business cycle. The empirical evidence I present in Section 2 demonstrates the presence of these compositional changes in unemployment flows. The model that follows shows that these can cause persistent unemployment independently of the human capital depreciation mechanism.

Recessions and True Duration Dependence vs. Unobserved Heterogeneity

Empirical reemployment probabilities decline with unemployment duration, and a significant literature has sought to understand the mechanism behind this. This pattern is consistent with the theoretical predictions of the selective hiring models described above, but human capital depreciation could also play a role.

More generally, this literature separates possible mechanisms into two categories: (1) true duration dependence (caused by human capital depreciation, the stigma of long-term unemployment, etc.) and (2) unobserved heterogeneity between the LTU and STU (which might be caused by selective hiring and negative sorting during unemployment). See Heckman (1991) and Machin and Manning (1999) for surveys of such analyses.

Several empirical papers focus more directly on the unemployment dynamics emphasized in my analysis; generally, these studies characterize variations over the business cycle in unobserved heterogeneity and in true duration dependence (Baker, 1992; Dynarski and Sheffrin, 1990; Kalwij, 2001; Imbens and Lynch, 2006).[6] Of these, Baker's analysis is most applicable to mine. He notes that unemployment durations increase overall during recessions, and he investigates how much of this rise results from changes in the composition of those entering unemployment.

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[6] Several somewhat related papers (such as Elsby, Michaels, and Solon, 2009 and Bachmann and Sinning, 2011) conduct more macro-oriented analyses of compositional changes in unemployment inflows and outflows. In particular, Bachmann and Sinning find that the compositional effects begin to decrease outflows from unemployment toward the end of a recession. As I will show in Section 2, this time period coincides with the shift from job leavers toward job losers, suggesting that this shift lowers the quality of flows to unemployment.

Baker finds that a significant part of the increase in durations is caused by changes specifically in the firings/quits composition of flows to unemployment.[7] This is certainly consistent with the evidence I present in Section 2, and this further supports the model I develop in the rest of the paper.

Lastly, Nakamura (2008) is especially relevant. She considers how and why flows to unemployment change during recessions, focusing specifically on changes in the quality of these flows. I discuss her conclusions and compare them to my own in the following section.

2. Changes in the Quality of Flows to Unemployment

2.1 The Standard View and the Importance of Job Leavers

"If you think about it, people who were laid off recently may be, on average, worse candidates than people who were laid off a while ago. After all, people who have been out of work for two years or longer are people who were laid off during the recession. That means many of them were workers whose jobs were eliminated simply because their businesses were doing badly, not because they were personally incompetent."

- Catherine Rampell, New York Times *Economix* Blog (July 26, 2011)[8]

Previous studies have either directly concluded (Nakamura, 2008) or implicitly suggested (Lockwood, 1991; Kosovich, 2010) that better workers enter unemployment during recessions. This is based on the following intuition: Workers generally have less value to their firms during a recession. As such, firms must raise standards for hiring new workers and for continuing to employ existing workers. Therefore, workers fired during recessions are of higher "quality" on average than those fired under other economic conditions. Extending this logic to the unemployment pool yields the conclusion that the quality of the unemployed rises during recessions.

Considering the aftermath of the most recent recession, this view is especially puzzling. Unemployment has remained high long after the recession's end, and recent firm hiring has benefitted mainly STU. In 2010, many firms began explicitly discriminating against LTU by

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[7]Baker's main argument is that compositional changes cannot explain all of the cyclical variation in unemployment durations, but he acknowledges that reasons for unemployment have a significant effect.

[8]URL: http://economix.blogs.nytimes.com/2011/07/26/discriminating-against-the-unemployed/

requiring that job applicants be either currently employed or recently laid off.[9,10,11] As the preceeding quote suggests, the continuing struggles of LTU are inconsistent with the standard view—those laid off during the recession should have disproportionate job-finding success if they are better workers. Instead, it seems almost as if firms have been trying to avoid specifically these workers.

Of course, the analysis behind this standard view ignores job leavers, and considering their role in employment dynamics can help us reconcile these inconsistencies. Two patterns support this possibility. First, the fractions of workers entering unemployment who are job leavers and job losers changes during recessions. At these times, the newly unemployed consist increasingly of job losers. Figure 1 shows this pattern during the most recent recession. The red and blue lines represent monthly quits and firings, respectively, according to JOLTS data.[12] The shaded region indicates the recession according to official NBER dates. Toward the end of the recession, there was a stark shift from job leavers toward job losers in the flows to unemployment.
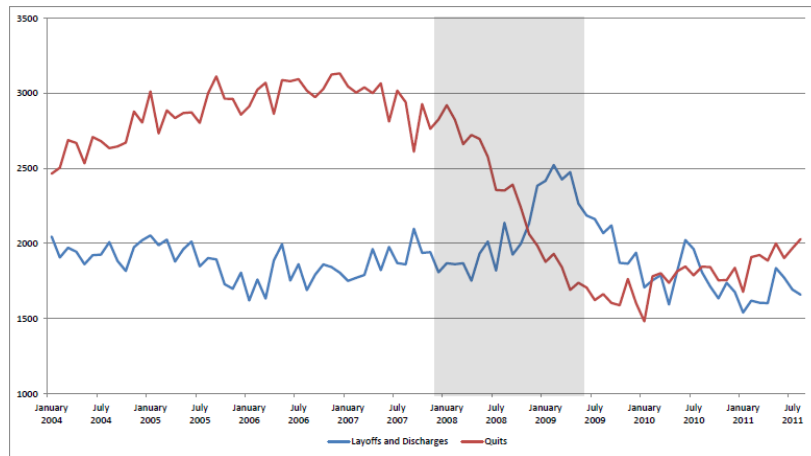

Figure 1: Evolution of firings vs. quits in the latest recession (Source: JOLTS)

Second, job losers take longer than job leavers to regain employment after entering the unemployment pool. This is supported in Figure 2 below, which separately plots the cumulative distribution functions of unemployment durations for job leavers (in red) and job losers (in

[9]Rampell, Catherine. 2011. "The Help-Wanted Sign Comes with a Frustrating Asterisk." The New York Times, July 25. URL:
http://www.nytimes.com/2011/07/26/business/help-wanted-ads-exclude-the-long-term-jobless.html
[10]National Employment Law Project. 2011. "Hiring Discrimination Against the Unemployed: Federal Bill Outlaws Excluding the Unemployed from Job Opportunities, as Discriminatory Ads Persist." Briefing Paper, July 12. URL: http://nelp.3cdn.net/b4ade339e970088d72_alm6blqx8.pdf
[11]"Jobless face rising discrimination." United Press International, Inc., February 17, 2011. URL: http://www.upi.com/Business_News/2011/02/17/Jobless-face-rising-discrimination/UPI-66711297965046/
[12]If these intensities are instead computed using CPS employment data, the patterns are virtually identical to those displayed here in the JOLTS data (which are based on firm responses). Hence, these trends are robust to the exclusion of job-to-job transitions from the data.

blue) using CPS employment data. Clearly, unemployment durations for job losers first-order stochastically dominate those for job leavers.
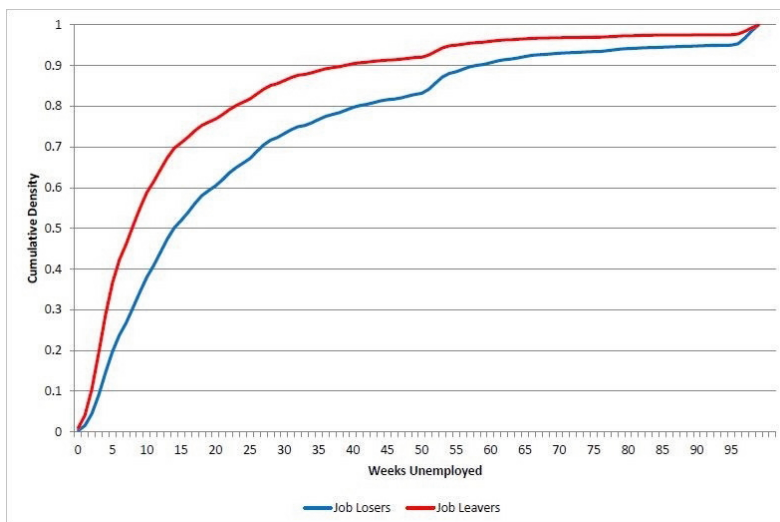

Figure 2: Unemployment Duration Distribution–Job Leavers vs. Job Losers (Source: CPS)

One might worry that the unemployment duration distribution for quits is skewed by those who quit with future employment already in place (and thus enter unemployment only for a brief time). Two factors address this concern. First, the duration of unemployment for fires first-order stochastically dominates the duration for quits even when both distributions are truncated below at various durations from 1 to 25 weeks of unemployment. Job leavers appear to have better reemployment prospects than job losers even among those who have already been unemployed for some time, so unemployed workers with future jobs in place cannot explain these distributional differences alone. Second, the data generating these distributions include only workers who were unemployed during the monthly census sampling date, so most workers who quit with another job already in place would not have remained unemployed long enough to enter these data.[13]

The superior reemployment prospects of job leavers suggest that these workers are more desirable to employers—that they are of better "quality." Given this, the shift toward job losers

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[13]Another concern about the implications of this pattern is the worry that, relative to fired workers, quitting workers disproportionately leave the unemployment pool by leaving the labor force, rather than by actually finding new jobs. Even if this is the case, though, unless there is positive selection among those who leave the labor force, this will not lower the perceived quality to firms of this pool of unemployed workers. In reality, there are many reasons to believe there is negative selection among those who leave the labor force (e.g. - if those who exit the labor force have received the most negative signals in the job market thus far, or if the perseverance required to continue job search also lends itself to performance on the job). If this selection is negative, then from an employer's perspective, this selection is actually improving the pool of job applicants who quit previous jobs. As such, this would be consistent with firms preferring workers who quit previous jobs to those who were fired.

during recessions could lower the quality of those entering unemployment. Of course, this quality depends not only on the relative shares of job losers and job leavers, but also on the qualities of these two groups. For instance, a shift from job leavers to job losers may not lower the quality of those entering unemployment if it is accompanied by a drastic rise in the quality of fired workers. The remainder of this section addresses such concerns.

2.2 A Test of the Mechanism

In what follows, I describe a test of the hypothesis that shifts from job leavers toward job losers result in lower quality workers entering the unemployment pool, and I present direct evidence supporting this.

Intuition

For the purposes of our analysis, we will separate this pool into two groups according to durations of unemployment. These groups will be divided at duration $T \in \mathbb{R}_+$—workers who have been unemployed for durations $\tau \in [0, T)$ are defined as STU, while those unemployed for durations $\tau \in [T, \infty)$ are LTU.[14] For clarity, this division is represented in Figure 3 below.
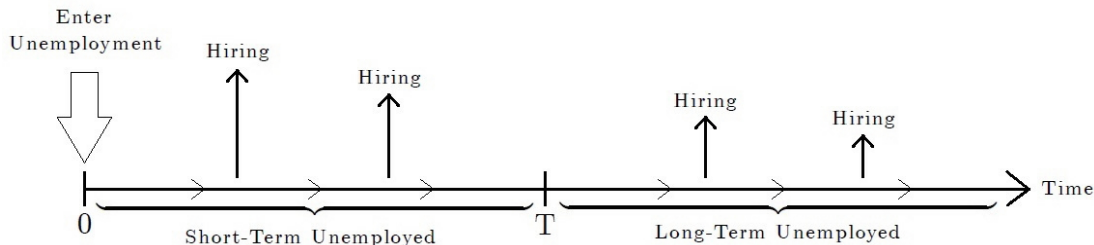

Figure 3: Unemployment Durations: Short- and Long-term Unemployed

To test for the quality of workers entering unemployment, we first must recognize that flows to unemployment are grouped initially with the STU. If workers entering unemployment fall in quality, this decrease in quality should first affect the STU; the LTU should see no change in quality until these workers have been unemployed for long enough to be classified in this group.[15] Thus, the short-term unemployed should initially worsen *relative* to the long-term unemployed. If hiring is selective, the reemployment probability of a given group should correlate with the quality of that group, so the reemployment probabilities of STU should also worsen relative to those of LTU.

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[14]In the empirical analysis that follows, I divide these duration groups between 12 and 13 weeks $\approx$ 3 months; individuals with unemployment durations of 1-12 weeks are STU, while those with durations of at least 13 weeks are LTU.

[15]This assumes that the quality of flows to unemployment at time t does not correlate perfectly with the quality of flows at time $t-T$. In other words, a drop in quality of new flows into the STU must not be perfectly cancelled by a simultaneous drop in the quality of flows from the STU to the LTU.

Given this reasoning, we have the following prediction: if increases in firings and decreases in quits lower the quality of flows to unemployment, then these changes should decrease the reemployment probabilities of the STU relative to those of the LTU.[16] Thus, we can use this to test whether shifts from firings toward quits lower the unemployment pool's quality.

Note that this result is informative specifically because it predicts different effects across durations of unemployment. For example, if we found that shifts from quits to firings lowered reemployment probabilities for the unemployment pool as a whole, this might simply reflect the fact that these shifts occur during periods when hiring decreases.[17] This would not, however, explain why the job-finding probabilities of the LTU improve relative to those of the STU.

Formal Predictions

To assess this result empirically, we must quantify how the reemployment probabilities of the short- and long-term unemployed vary in response to changes in the fires/quits composition of flows to unemployment. Toward this end, let us first formalize the prediction we want to test. Define H_t^S to be the probability that STU in period t are reemployed by period $t+1$, and define $H_t^{;L}$ to be the corresponding probability for LTU. In turn, let H_t represent this probability for the set of all unemployed across both groups. Further, define Q_t to be the number of quits at time t, scaled by the size of the unemployment pool, and define F_t to be the corresponding scaled value for firings at time t. Thus, these can be written as

$$H_t^S \equiv \frac{\text{\# of STU in period } t \text{ hired by period } t+1}{\text{\# of STU in period } t}$$

$$H_t^L \equiv \frac{\text{\# of LTU in period } t \text{ hired by period } t+1}{\text{\# of LTU in period } t}$$

$$Q_t \equiv \frac{\text{\# of quits in period } t}{\text{\# of unemployed in period } t}$$

$$F_t \equiv \frac{\text{\# of firings in period } t}{\text{\# of unemployed in period } t}$$

Restating our goal in the newly established notation, we want to determine how H_t^S and H_t^L respond to Q_t and F_t. We can characterize these relationships in terms of four elasticities: $\frac{\partial \ln(H_t^S)}{\partial \ln(F_{t-1})}$, $\frac{\partial \ln(H_t^L)}{\partial \ln(F_{t-1})}$, $\frac{\partial \ln(H_t^S)}{\partial \ln(Q_{t-1})}$, and $\frac{\partial \ln(H_t^L)}{\partial \ln(Q_{t-1})}$. Time lags are included between firings/quits and hiring because, as will be explained below and in Appendix C, hiring outcomes are drawn from monthly CPS data. These lags ensure that flows to unemployment are counted among the STU when determining reemployment probabilities. An individual's first appearance in unemployment (in the data) might correspond—in reality—to his first or second week of joblessness. In

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[16] Using a mechanical model of the unemployment pool's evolution, it is straightforward to show formally that this prediction holds under very weak assumptions.

[17] In the empirical analysis to follow, we also control for changes in hiring intensity.

such cases, high quality unemployed individuals have little time to distinguish themselves by re-claiming employment, so the effects of changes in the quality of flows to unemployment might be muted empirically. Because I group individuals among the STU during their first three months of joblessness, we can better detect changes in quality by introducing 1-2 months of lag to this estimation.

Then we can write the prediction that increases in firings should lower the reemployment probabilities of STU relative to those of LTU as

$$\frac{\partial \ln \left(H_t^S \right)}{\partial \ln \left(F_{t-1} \right)} - \frac{\partial \ln \left(H_t^L \right)}{\partial \ln \left(F_{t-1} \right)} < 0$$

and we can write the prediction that increases in quits should have the opposite effect as

$$\frac{\partial \ln \left(H_t^S \right)}{\partial \ln \left(Q_{t-1} \right)} - \frac{\partial \ln \left(H_t^L \right)}{\partial \ln \left(Q_{t-1} \right)} > 0$$

Data and Results

Here, I will briefly describe the data used to assess these predictions—for a complete description of the data and empirical methodology, see Appendix C. Hiring outcomes of the unemployed are drawn from individual-level CPS monthly employment data. For robustness of the results, flows of firings and quits come from two independent sources: (1) I calculate these values directly using the monthly CPS data[18] and (2) I use monthly JOLTS aggregate data to obtain a second set of these values.

Both sources are imperfect with regard to this specific analysis: JOLTS data reflect firm reports of labor turnover, and these figures may include job-to-job transitions, which do not involve the unemployment pool itself. If the intensity of these transitions fluctuates more or less than the inflows to and outflows from unemployment, then this may be a noisy (or even biased) representation of unemployment flows. In contrast, the CPS surveys individual workers, so it allows exclusion of those who never enter unemployment. Unfortunately, we can detect those in the unemployment pool only if they are in this pool at the time of the monthly survey, so the CPS may underestimate the relevant flows. To deal with these concerns, I obtain the results that follow using each of these data sources separately.

The sample used is restricted to men with no more than a high school education—this is merely to focus on the population groups where the mechanism has the most consistent effects. The relevant results persist when women and other education groups are included, but estimates obtained are less precise.

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[18] I measure flows of firings and quits by totalling unemployed job leavers and losers with unemployment durations in the range of 1-4 weeks. I calculate the intensity of hiring as the fraction of unemployed workers who are successfully linked to the next month who gain employment in this period. (The fraction of the sample unable to be matched was extremely small—see Appendix C and Rothstein (2011) for more detailed discussions of the process of matching consecutive months in the CPS).

Specifically, women are excluded to avoid the complications caused by weak labor market attachment (such as a greater willingness to respond to adverse shocks by substituting effort from the labor market toward family investment). In turn, those without higher education faced the steepest increases in unemployment incidence during the recession, so the causes of this group's rising unemployment are crucial to understanding aggregate employment dynamics. Further— regarding the mechanism suggested in this paper—education is a tool for signaling competence to prospective employers, so those with lower educational attainment may be less able to distinguish themselves from the unemployment pool. Therefore, increases in targeted firings may impact this group's reemployment probabilities more severely than others. Consistent with this, the unemployment rate among those without a high school diploma is more than twice as great as that among those with greater educational attainment. Further, during the recession, this unemployment rate for non-high school graduates increased more than twice as much as that for higher educational attainment groups.

Further, because firings and quits are central to this compositional change mechanism, the sample used includes only job losers and job leavers. However, the additional inclusion of re-entrants and new entrants to the labor force has no discernable effects on the results.

Using the data described above, Table 1 confirms the aforementioned predictions:

Table 1: Responses of STU - LTU hiring probabilities to firings/quits (Sources: CPS, JOLTS)

Source of unemployment flows data:	(I)		(II)		(III)		(IV)	
	CPS	JOLTS	CPS	JOLTS	CPS	JOLTS	CPS	JOLTS
$\frac{\partial \ln\left(H_t^S\right)}{\partial \ln\left(F_{t-1}\right)} - \frac{\partial \ln\left(H_t^L\right)}{\partial \ln\left(F_{t-1}\right)}$	-0.056	-0.071	-0.062	-0.077	-0.043	-0.079	-0.018	-0.059
	(0.072)	(0.097)	(0.074)	(0.102)	(0.071)	(0.097)	(0.066)	(0.089)
$\frac{\partial \ln\left(H_t^S\right)}{\partial \ln\left(Q_{t-1}\right)} - \frac{\partial \ln\left(H_t^L\right)}{\partial \ln\left(Q_{t-1}\right)}$	0.137	0.288	0.141	0.233	0.141	0.237	0.129	0.135
	(0.051)	(0.183)	(0.052)	(0.184)	(0.054)	(0.166)	(0.053)	(0.154)
Probability under null	0.007	0.041	0.005	0.069	0.009	0.048	0.022	0.133
Control for $\partial \ln\left(H_t\right)$?	Y	Y	N	N	Y	Y	Y	Y
Use sampling weights?	Y	Y	Y	Y	N	N	Y	Y
Condition on individual observables?	Y	Y	Y	Y	Y	Y	N	N
N	54,059	54,059	54,059	54,059	54,059	54,059	54,059	54,059

Clustered standard errors (by year-month) in parentheses
Data used above include unemployed men with no education beyond a HS diploma over the period Jan 2001 to Aug 2008

In all specifications, increases in firings worsen the reemployment probabilities of the STU relative to those of LTU. Similarly, increases in quits have the opposite effect in all specifications. For detailed descriptions of how these specifications differ, see Appendix C.

Additionally, the highlighted section of the table displays the results of tests of the aforementioned predictions. For simplicity, these predictions have been summarized into a single hypothesis test regarding the effect of shifts from quits toward firings. Specifically, we test the

hypothesis that these shifts improve the relative reemployment probabilities of the STU:

$$\frac{\partial \ln\left(H_t^S\right)}{\partial \ln\left(Q_{t-1}\right)} - \frac{\partial \ln\left(H_t^L\right)}{\partial \ln\left(Q_{t-1}\right)} > \frac{\partial \ln\left(H_t^S\right)}{\partial \ln\left(F_{t-1}\right)} - \frac{\partial \ln\left(H_t^L\right)}{\partial \ln\left(F_{t-1}\right)}$$

against the null that

$$\frac{\partial \ln\left(H_t^S\right)}{\partial \ln\left(Q_{t-1}\right)} - \frac{\partial \ln\left(H_t^L\right)}{\partial \ln\left(Q_{t-1}\right)} = \frac{\partial \ln\left(H_t^S\right)}{\partial \ln\left(F_{t-1}\right)} - \frac{\partial \ln\left(H_t^L\right)}{\partial \ln\left(F_{t-1}\right)}$$

There is consistent support for rejecting this null hypothesis across specifications and data sources—we can reject the null at the 5% level (or lower) in all but one of the cases displayed. In the one exception to this (specification IV using JOLTS data), the estimated effects of firings and quits are still consistent with the given predictions. The higher p-value is obtained because individual-level covariates are not used, so the resulting estimates are less precise.

Thus, we have robust evidence that shifts from quits to firings result in lower quality workers entering unemployment.

Implications for Recessions

The analysis thus far was intended to detect the compositional changes that accompany moderate economic fluctuations. However, the theoretical analysis in the following sections characterizes the consequences of these changes in response to a significant economic downturn, so it will be useful to see whether these results extend to recessions. For this purpose, we now investigate the evolution of differences between the hiring outcomes of STU and LTU during the recent recession.[19]

Note that the data used in Table 1 are restricted to the period January 2001 - August 2008. For September 2008 - August 2011, the corresponding estimates are similar, but magnified. This is because changes in the flows of firings and quits are serially correlated during sustained economic fluctuations—during a downturn, firings will rise and quits will fall in consecutive months. Because I group individuals among the short-term unemployed during their first 3 months of joblessness, these sustained shifts will compound the effects of several months of compositional changes, and estimated reemployment disparities between the short- and long-term unemployed will be larger. In this sense, the estimates in Table 1 are weakened by the short-term unemployed who remain from the previous month; the effects are generated only by those new to the unemployment pool.

Corresponding estimates for the periods January 2001 - August 2011 and September 2008 -

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[19]To study this recession, we must restrict ourselves to a smaller, noisier data sample. As such, this section focuses on characterizing patterns over time and checking that these are consistent with the mechanism's predictions (rather than formal hypothesis testing).

August 2011 are given in Table 2 (in Appendix C).[20] The logic given above suggests that the estimated effects should be stronger in the full sample than in the pre-August 2008 data. In turn, these estimated effects should be stronger still in the post-August 2008 sample. This is precisely what we observe; for the full time period, we estimate $\frac{\partial \ln\left(H_t^S\right)}{\partial \ln(F_{t-1})} - \frac{\partial \ln\left(H_t^L\right)}{\partial \ln(F_{t-1})}$ to be -0.101 and $\frac{\partial \ln\left(H_t^S\right)}{\partial \ln(Q_{t-1})} - \frac{\partial \ln\left(H_t^L\right)}{\partial \ln(Q_{t-1})}$ to be 0.811. For the September 2008 - August 2011 period, these estimates grow in magnitude to -0.471 and 1.242.

These time periods are omitted from Table 1 for precision—because the serial correlation in flows is not present in all months of the data, the measured effects will vary across months. As a result, estimates obtained using data for the entire time period will be noisier than those in Table 1.[21] Accordingly, the (more precise) estimates in Table 1 can be viewed as lower bounds for the effects of compositional changes on hiring outcomes.

Additionally, there was a sustained shift toward firings during late 2008 and early 2009, and the model developed in this section suggests that the reemployment probabilities of the LTU should have risen significantly in comparison to those of the STU. It is important to note that this relative improvement would be brief—the lower quality workers entering unemployment would initially lower the relative outcomes of STU, but they would become LTU after 13 weeks and would depress the outcomes of this group thereafter. Figure 4 below displays the probabilistic hiring advantage of STU over six month intervals through the recession (the monthly flows of firings and quits appear as well).

Figure 4: Hiring likelihood advantage of STU through the recession (Sources: CPS, JOLTS)

[20]Having already established consistency between the CPS-based and JOLTS-based estimates for the earlier sample, I report the (more precise) estimates using CPS-based firings/quits in this case.

[21]One might worry about similar problems being caused by the firings/quits dynamics around the 2001 recession. The findings in both Table 1 and Table 2 (where applicable) are robust to the exclusion of this time period. The results persist—both in magnitude and in precision—for arbitrary sample start dates in the 2000-2004 range.

It is clear that this sustained shift toward firings was accompanied by a sharp, relative decline in reemployment likelihood for STU. Indeed, this decline had disappeared by the following six-month interval (and many of those who lost jobs during the surge of firings had joined the group of LTU by the following six-month period). Further, this disparity is statistically robust. During the 6 month interval when the advantage of STU is smallest (14.3%, which is 3.5% - 6% below the corresponding advantages for other intervals shown), the standard error of this advantage is 0.771%, so it is unlikely that this decline can be explained by empirical noise.

Motivated by this evidence, we begin the theoretical analysis below.

3. The Model

The economy consists of a unit measure of workers and a mass of firms determined by free entry. Time is continuous and infinite, and firms discount the future at rate $r > 0$. All firms are identical, but workers are distinguished by a type $\theta \in \{H, L\}$. Of the unit measure of workers in the economy, the proportion $Q \in (0, 1)$ are type H.

Employment, Payoffs, and Wages

Firms make hiring and firing decisions—each can employ at most 1 worker at a time and must pay the instantaneous flow cost w to do so. Firms receive a payoff $Y > 0$ with Poisson intensity $\lambda > 0$ from each type H worker, and they receive no payoffs from type L workers. In turn, workers face a binary choice between working for wage rate w and unemployment, which offers a value normalized to 0. To be employed, workers have the reservation value $\widetilde{w} \in (0, \lambda Y)$, which is known to both workers and firms.

It is important to note that the model admits several interpretations of $\widetilde{w}$. Viewed as a reservation wage, $\widetilde{w}$ could represent the flow effort cost of labor or even a government-imposed minimum wage. Alternatively, we need not even consider $\widetilde{w}$ to be compensation for workers—it could represent a flow cost of operation for the firm. (In this case, the worker's actual wage would be $w - \widetilde{w}$). Yet another possibility, in the spirit of Ramey and Watson (1997), is for $\widetilde{w}$ to reflect a payoff the worker can obtain by misbehaving (such as stealing or destroying property) and leaving the job immediately. In this case, $\widetilde{w}$ is the wage at which the worker's present value of the employment relationship is equal to the value he can obtain by misbehaving and quitting.

The only difference between these cases is the source of the firm's production cost; from the firm's perspective, they are otherwise identical. While these distinctions may matter in welfare analysis, the interpretation of $\widetilde{w}$ will not impact equilibrium labor market outcomes and dynamics. For consistency, the remainder of the paper will simply refer to $\widetilde{w}$ as the reservation wage.

At Poisson intensity $\pi > 0$, workers voluntarily quit their jobs and return to the unemployment pool (in search of new jobs). This simple, exogenous shock is meant to represent a personal reason for wanting to leave, such as a need to move geographically for family reasons or a developing distaste for the tasks of the current job. We assume that this motivation for leaving is strong enough that the firm cannot profitably retain the worker at a renegotiated, higher wage.

Hiring and Contracts

When out of the market, firms must pay cost $c > 0$ to hire a worker from the unemployment pool. This cost c can be interpreted as including the search/interview/hiring costs associated with obtaining a new employee. Firms cannot observe specific worker types before hiring. However, because all firms are ex ante identical (and thus have identical equilibrium strategies), firms can infer the fraction of the unemployed at time t with $\theta = H$.

Firms can use a *screening technology* to refine the pool of potential hires, where the cost of screening is included in the hiring cost c. For each firm, this screening technology instantly filters the unemployment pool into a hiring pool—type H workers pass through this filter with probability 1, while type L workers pass through with probability $\alpha \in [0,1]$. Thus, if the unemployment pool has proportion q_U of type H workers, the hiring pool after screening will have the type H proportion $q_H(q_U) = \frac{q_U}{q_U + (1 - q_U)\alpha}$.

It is worth emphasizing here that this screening technology reflects *all* individual characteristics observable to the firm. Thus, within the model, it does not make sense for firms to condition explicitly on worker characteristics, such as unemployment duration, reason for unemployment, etc. Insofar as these attributes are observed, firms condition on them implicitly—they are already used in the screening technology to improve the chances of hiring a type H worker.[22]

Firms offer workers a fixed-wage contract, which will pay the flow value w at each instant while the worker is employed—firms reserve the right to terminate employment. As we will focus on employment (rather than on wages and contracting), we will assume that the realization of output Y is not observable to workers and is not contractible regardless. The wage level w is determined competitively; firms will attempt to outbid each other until it is no longer *ex ante* profitable to do so.

Learning and Employment Termination

Over time, firms learn about worker quality through payoff realizations. Suppose that, at time t, a firm has belief p_t about the probability that its employee is type H. Firms update according to Bayes' rule; if the firm receives a payoff at time t, it updates discretely to $p_{t+dt} = 1$,

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[22]This differs from Lockwood (1991), for instance, where hiring firms are able to condition on employment histories in addition to screening.

as this could not have happened with a type L worker. In the absence of a payoff at time t, the firm shifts its belief infinitessimally downward by $dp_t = -\lambda p_t (1 - p_t) dt$.

Matches end either when workers quit or when firms decide to fire them. To understand when firing is optimal for the firm, note that it faces three options at each point in time: (1) It can retain its current employee, offering value $V(p_t)$. (2) It can fire its current employee and leave the market, offering value 0. (3) It can fire its current employee and pay cost c to hire a new worker by applying the screening technology to the unemployment pool. This offers value $V\left(q_H\left(q_{U(t)}\right)\right) - c$, where $q_{U(t)}$ is the proportion of type H workers among the unemployed at time t.[23]

The firm will always choose optimally among these, so (taking the wage as given) it has the value function:

$$V(p_t) = \max\left\{ [\lambda p_t Y - w] \, dt + e^{-rdt} \mathbb{E}\left[V(p_t + dp_t)\right], \ 0, \ V\left(q_H\left(q_{U(t)}\right)\right) - c \right\}$$

Obviously, the current match's value is increasing in the belief p_t, so the firm will make termination decisions according to a threshold rule. It will prefer to end the relationship via option (2) or (3) when its belief about its worker's type falls to some p^*, at which point it can obtain equal value either from leaving the market or from hiring a replacement.

To characterize the firm's optimal firing decisions (and hiring decisions) in more detail, we must determine how the outside option evolves in equilibrium. This outside option depends on the value of hiring, which is affected by aggregate labor market conditions. The next section will characterize these conditions, so a more precise discussion of firm behavior will be included at that point.

4. Steady-State Equilbrium

4.1. Full-Information Outcome

Before analyzing this economy, we will digress briefly to consider a labor market with full-information. Suppose that employers can perfectly observe worker types before hiring, so there is no role for learning during employment.[24] Assume for convenience that $V(0) < 0$ and $\lambda Y - \widetilde{w} \geq 0$, so that firms will either employ type H workers or leave the market. Type H workers will never be fired intentionally, but they can still reach the unemployment pool by quitting. Free

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[23]Note: after a match has been terminated, firms must pay c to hire a new worker regardless of whether the termination was targeted or exogenous. In this sense, we exclude firing costs from those represented by c. Adding a parameter to capture these firing costs has no substantive impact on the model's main qualitative predictions.

[24]Note that this full-information setting can be viewed as the exteme case in which the screening technology's effectiveness parameter $\alpha = 0$.

entry will force wages for type H workers up to $w = \lambda Y - (r + \pi) c$, so that the value of hiring a type H worker is 0. As such, high type workers who reach the unemployment pool will be instantly hired at this wage. Thus, employment (which we denote by E) will be $E = Q$.

In the remainder of this section, we will show how the above outcome changes in a setting with *ex ante* uncertainty and learning. We will see that these forces can hinder employment even in the absence of search frictions. In order for hiring to occur, wages must be significantly lower to compensate firms for the risk of hiring a low type worker. Depending on the reservation value $\widetilde{w}$, the steady-state employment level can be lower or even higher than Q. Of course, it may be misleading to compare these two cases based on employment levels alone; under *ex ante* uncertainty, firms will sustain employment only at wages well below those paid under full information.

4.2. Equilibrium with Learning

The remainder of this section will characterize properties of equilibrium when firms cannot perfectly separate worker types before hiring. This will serve as a conceptual starting point for the following section's analysis of employment dynamics during and after recessions. We will therefore focus on equilibria that are economically meaningful and well-suited to this application, and these equilibria will satisfy three restrictions. *We will assume the following conditions are satisfied throughout the remaining analysis.*

First, in order to highlight the implications of learning during employment, we will limit the amount of ex ante information that firms can acquire before hiring.[25] Translating this to the model, we will rule out α that are too close to 0. Formally, if we define q_H^* to be the belief level at which the firm's value function is equal to the hiring cost c, then this assumption can be written in terms of parameters as:

$$\alpha > \left(\frac{r + \pi}{r + \pi + \lambda} \right) \left(\frac{\widetilde{w}}{\lambda Y - \widetilde{w}} \right) \left(\frac{1 - q_H^*}{q_H^*} \right) \tag{A1}$$

Often, no steady-state equilibrium exists for parameter combinations that violate this condition. If equilibria do exist, they will have undesirable properties for our applications, such as prohibitively high wages and rapid turnover. Most importantly, the type H proportion among fired workers (p^*) will be higher than that in the unemployment pool (q_U);[26] if firms could hire exclusively from workers fired by other firms, they would prefer to do this. Firings must *improve* the unemployment pool—this conflicts both with basic intuition and with the results presented in Section 2.

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[25] Intuitively, we want to distinguish this analysis from the full information benchmark, which corresponds to $\alpha = 0$.

[26] This is demonstrated in Appendix A in the proof of Proposition 4 (which will be presented in the next section).

Our second restriction will rule out parameter values that lead to zero employment in equilibrium. We will do this by placing a lower bound on Q, the proportion of type H workers in the labor force. There can be no positive selection into unemployment, so the unemployment pool's quality is bounded above by Q. Given values for the subset of parameters $\{\alpha, r, \pi, \lambda, Y, \widetilde{w}\}$, if Q is too low, then the firm will negatively value a worker at belief $q_H(Q)$. In equilibrium, firms will never be willing to hire, even at a hiring cost of 0. Formally, for α values that satisfy **(A1)**, we require a sufficiently high labor force quality:[27]

$$Q > \frac{\alpha (r + \pi) \widetilde{w}}{(\lambda Y - \widetilde{w})(r + \pi + \lambda) + \alpha (r + \pi) \widetilde{w}} \tag{A2}$$

Lastly, we will restrict our focus to "nontrivial steady-state employment equilibria," which must satisfy the following definition:

Definition 1: *A **steady-state employment equilibrium** $\left\{ w_t^*, p_t^*, E_t, q_{U(t)}, q_{E(t)} \right\}_{t=0}^{\infty}$ consists of market wage rates w_t^*, threshold rules $p_t^* \in [0, 1]$, employment levels $E_t \in [0, 1]$, and type H proportions of unemployed workers $q_{U(t)} \in [0, 1]$ and employed workers $q_{E(t)} \in [0, 1]$ such that: (i) firm firing decisions are optimal, (ii) firm hiring decisions are optimal (free entry: $V\left(q_H\left(q_{U(t)}\right)\right) \leq c$), (iii) the size and quality of the unemployment pool are consistent with the size and quality of the total labor force, and (iv) the employment level is constant ($E_t = E_{ss}, \forall t$).*

*Further, such an equilibrium is **"nontrivial"** if $E_{ss} \in (0, 1)$.*

Note that Proposition 1 (in Appendix A) establishes the existence of a unique nontrivial steady-state employment equilibrium for a range of "reasonable" hiring costs c. It is also shown in Appendix A that this result holds when **(A1)** and **(A2)** are satisfied.

Several aspects of this definition merit discussion. Practical reasons motivate our restriction to equilibria that are nontrivial and meet Condition (iv). Condition (iv) is for simplicity—by focusing on a steady-state, we can provide clear intuition for the results in this section and the next. Additionally, it is worth noting the existence of equilibria which fail this condition. In these cases, employment fluctuates *even without exogenous shocks*. Though these equilibria are not used in this section's analysis, they highlight more general conceptual insights regarding the structure of information in labor markets. I discuss this briefly in Section 6.

In turn, nontrivial equilibria in this model are those that are economically relevant. In a steady-state, full employment equilibria have little insight to offer about unemployment. More importantly, to study unemployment's dynamic response to economic shocks, we require a model in which unemployment is present not only after the shock, but also in the pre-shock equilibrium.[28]

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[27]Note that we could also express this as a lower bound on Y or as an upper bound on $\widetilde{w}$, among other things.

[28]It should become clear in this section that this is merely a constraint on extreme parameter values, rather than a conceptual restriction on the economy.

The remaining aspects of this definition—the first three conditions—are general properties of the economy. Condition (i) requires value matching $(V(p_t^*) = 0)$ and smooth pasting $(V'(p_t^*) = 0)$, where the latter must hold because downward updating of firm beliefs p_t is differentiable. Condition (iii) is mainly one of accounting—the measure of type H workers in the labor force is constant, so this must equal the sum of the measures of type H workers among the employed and unemployed. Formally, this requires $E_t q_{E(t)} + (1 - E_t) q_{U(t)} = Q$.

Condition (ii), free entry, simply implies that firms cannot make *ex ante* profits from hiring. Like conditions (i) and (iii), this is also a standard property, but it is crucial to understand how exactly firms compete away these profits in equilibrium. The free entry condition is maintained through three channels in the model:

Scarcity: If there is full employment in the economy, and if the new workers entering unemployment are of sufficiently high quality, they will be rehired immediately. Firms will be unable to profit from employment because there are no workers to hire.

Scarcity can be relevant only if employment is full, so it will be ignored in the remainder of this analysis. The next two mechanisms, however, are also important for equilibria with unemployment.

Competitive Wage Bidding: Suppose that the wage level is w, and suppose also that the unemployment pool quality is sufficiently high that $V(q_H(q_{U(t)})) > c$ at these wages. Firms will compete to profit from workers, and in doing so, they will bid up wages until these profits have been eliminated (and $V(q_H(q_{U(t)})) = c$).

Selective Hiring: In using the screening technology, firms disproportionately remove type H workers from the unemployment pool. When one firm hires a worker, it marginally lowers the quality of the unemployment pool from which other firms must hire, and in turn, it lowers the expected value of a new hire as well. Thus, in addition to raising wages, firms can compete away profits by hiring more intensely.

In the analysis to follow, we will see that selective hiring is dominant among these forces in equilibrium. Toward demonstrating this, it will be useful to establish several properties of nontrivial steady-state employment equilibria. First, note that there is no intrinsic heterogeneity among firms, so the equilibrium wage and threshold rules characterize the behavior of all firms.

It is straightforward to show that wages must equal the reservation value:

Lemma 1: *In any nontrivial steady-state employment equilibrium, $w = \widetilde{w}$.*

(See Appendix A for proof)

Intuitively, if firms were offering wages greater than $\widetilde{w}$, unemployed workers could benefit from undercutting these offers and working for less; thus, this cannot occur in equilibrium.

With this result, we can also show that all aggregate characteristics of the economy must be fixed in equilibrium. More specifically, define η_t to be the intensity of hiring at time t, so $\eta_t dt$

is the measure of unemployed workers hired at this instant. Then we can write our aggregate values as $\eta_t = \eta_{ss}$, $q_{U(t)} = q_{U(ss)}$, and $q_{E(t)} = q_{E(ss)}$.[29]

Another crucial implication is that the free entry condition must hold with equality: $V\left(q_H\left(q_{U(ss)}\right)\right) = c$. Let us define q_U^* to be the minimum unemployment pool quality at which firms are willing to hire at reservation level wages. q_U^* must satisfy $V\left(q_H\left(q_U^*\right)\right) = c$ (and $q_H^* = q_H\left(q_U^*\right)$), so the equilibrium unemployment pool quality is $q_{U(ss)} = q_U^*$. This is why wage bidding effects are suppressed—firms cannot offer wages above $\widetilde{w}$ because the expected value of hiring is negative at such wages.

Because of selective hiring, the labor market is stable at this equilibrium. To see this, consider first an equilibrium without selective hiring in which there is positive unemployment. With the unemployment pool of quality q_U^*, firms are perfectly indifferent between hiring and not. There exists a constant intensity of firm hiring such that the unemployment pool quality and size are unchanged, but there is no reason for this intensity to be realized. These conditions can be sustained only if we assume that precisely the right measure of indifferent firms choose to hire at each instant.

In contrast, when hiring is selective, the aggregate intensity of hiring directly impacts the value of hiring. If too few firms were hiring, the unemployment pool quality would rise and more firms would find it profitable to hire. If too many firms were hiring, their screening process would deplete the quality of the unemployment pool, and it would no longer be profitable to hire. Thus, the economy will support precisely the intensity of hiring such that the unemployment pool remains at quality q_U^*.

We will discuss this in more detail at the end of this section. At that point, we will analytically characterize the equilibrium forces acting on the unemployment pool. First, though, we digress briefly to explain how these forces are generated by optimal firm decisions.

Equilibrium Firm Behavior

Free entry has another consequence in this setting—firms always face an outside option of value 0. In conjunction with the unvarying equilibrium wage level, this implies that the firm value function depends only on the belief p_t. We can thus use value-matching and smooth-pasting conditions to solve *explicitly* for the firm's value function:[30]

Proposition 2: *In a nontrivial steady-state employment equilibrium, the firm's value function can be written analytically as:*

$$
V_{ss}\left(p_t\right) = \begin{cases} \left(\frac{1}{r+\pi}\right)\left[\lambda p_t Y - \widetilde{w} + \lambda \left(\frac{\widetilde{w}(1-p_t)}{r+\pi+\lambda}\right)^{\frac{r+\pi+\lambda}{\lambda}}\left(\frac{(\lambda Y - \widetilde{w})p_t}{r+\pi}\right)^{-\frac{r+\pi}{\lambda}}\right] & \textit{for } p_t \in [p_{ss}^*, 1] \\ 0 \textit{ for } p_t \leq p_{ss}^* \end{cases}
$$

---

[29] This is proven after Lemma 1 in Appendix A.

[30] Insights from Bellman and Cooke (1963), Presman (1990), and Keller, Rady, and Cripps (2005) are used in the derivation of this value function.

Further, this threshold level is given by $p^*_{ss} = \frac{\widetilde{w}(r+\pi)}{\lambda[Y(r+\lambda+\pi)-\widetilde{w}]}$

(See Appendix A for proof)

As can be seen in the expression for p^*_{ss}, firms will retain workers at lower beliefs when they face higher payoffs Y, faster learning (and more frequent payoffs) λ, less frequent worker quits π, less firm "impatience" r (or lower interest rates), and lower wages $\widetilde{w}$.

Additionally, the value function itself illustrates the role of learning in this setting. The term $\frac{\lambda p_t Y - \widetilde{w}}{r+\pi}$ represents the employee's expected output, as a function of the perceived likelihood that she is type H. A "myopic" firm would consider this value alone. It would employ only workers offering an expected instantaneous profit, so it would terminate employment at the belief $p^*_m = \frac{\widetilde{w}}{\lambda Y}$.

However, each firm can terminate workers at its own discretion. As a result, the firm has an option value of eliminating an unproductive worker and either hiring a new worker or leaving the market. In the value function above, this option value is captured in the additive term on the right, which is clearly decreasing in the belief p_t. Intuitively, a decline in p_t decreases the value of the current employee and increases the likelihood that the firm will want to use the outside option. Since this outside option is fixed in value, its relative value increases.

Due to the option to drop unproductive employees, optimal firms are willing to retain employees with negative expected flow values; this is why $p^*_{ss} < p^*_m$. Figure 5 below reflects this intuition, comparing myopic and optimal value functions over the range of beliefs $p_t \in [p^*_{ss}, 1]$.
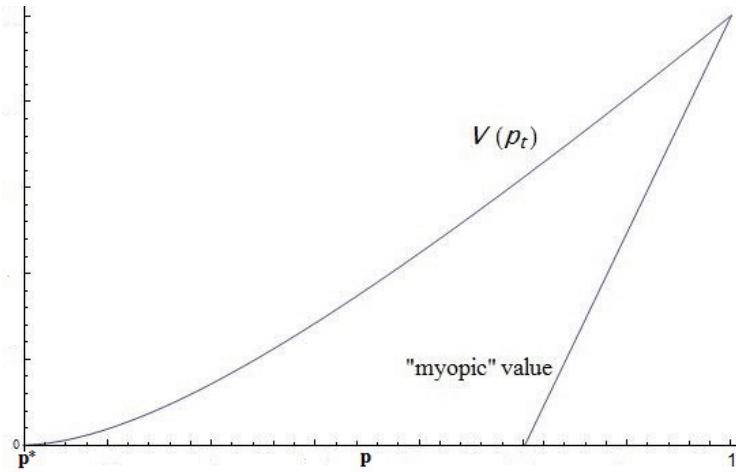

Figure 5: Firm value as a function of belief p_t

Extending this optimal behavior throughout the labor market, we can characterize the equilibrium distribution of firm beliefs, which will appear (approximately) as below:
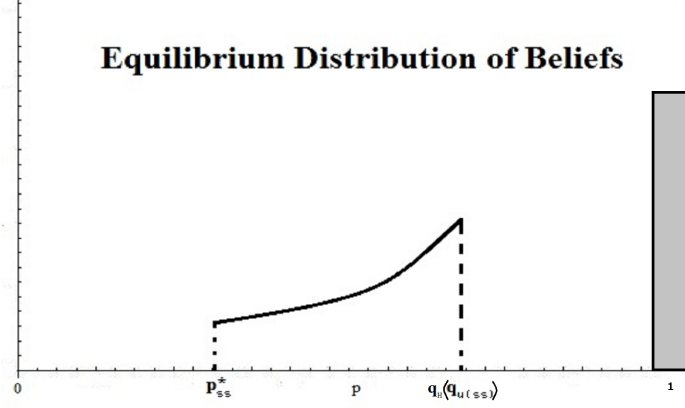
Figure 6: Distribution of Firm Beliefs

Because firms apply the screening technology to the unemployment pool, they can expect a new hire to be type H with probability $q_H\left(q_{U(ss)}\right)$. While this individual is employed, the firm's belief p_t declines gradually unless the firm recieves a payoff Y—if this occurs, the firm knows its worker must be of type H, so it updates to $p_{t+dt} = 1$ and employs the worker until random termination of the match (via π). Thus, the mass at $p_t = 1$ corresponds to firms who know they have type H employees. Without a payoff, the firm will employ its worker until its belief falls to p_{ss}^*, at which point the firm will terminate the match intentionally and either hire a new worker or leave the market.

We can think of the time after hiring required for firm beliefs to reach p_{ss}^* as a reflection of how "patient" firms will be with unproductive workers. This time (which we denote by t_{ss}^*) can be written as:

$$t_{ss}^* = \frac{1}{\lambda} \ln\left[\left(\frac{1 - p_{ss}^*}{p_{ss}^*}\right)\left(\frac{q_{U(ss)}}{\alpha\left(1 - q_{U(ss)}\right)}\right)\right]$$

Proposition 3 (relegated to Appendix A) provides this, as well as expressions for the equilibrium hiring intensity η_{ss} and employment level E_{ss}.

Steady-State Unemployment Inflows and Outflows

We can use the above belief distribution to characterize the forces acting on the unemployment pool's quality in equilibrium. With constant hiring and employment, the quality of this equilbrium unemployment pool will evolve according to:

$$q_{U(t+dt)} = \frac{\overbrace{[1 - E_{ss}]\, q_{U(t)}}^{\text{unemployed}} \overbrace{-\eta_{ss} dt q_H\left(q_{U(t)}\right)}^{\text{hirings}} + \overbrace{\eta_{ss} dt q_H\left(q_{U(t-t_{ss}^*)}\right) e^{-(\lambda+\pi)t_{ss}^*}}^{\text{directed firings}} + \overbrace{\pi dt E_{ss} q_{E(t)}}^{\text{quits}}}{\underbrace{1 - E_{ss}}_{\text{unemployed}} \underbrace{-\eta_{ss} dt}_{\text{hirings}} + \underbrace{\eta_{ss} dt \left[\left[1 - q_H\left(q_{U(t-t_{ss}^*)}\right)\right] e^{-\pi t_{ss}^*} + q_H\left(q_{U(t-t_{ss}^*)}\right) e^{-(\lambda+\pi)t_{ss}^*}\right]}_{\text{directed firings}} + \underbrace{\pi dt E_{ss}}_{\text{quits}}}$$

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where clearly we require $q_{U(t+dt)} = q_{U(t)} = q_{U(ss)}$ (and $q_{E(t)} = q_{E(ss)}, \forall t$) since the quality is constant in this equilibrium. To focus on the specific sources of unemployment flows in the equation above, we will continue writing $q_{U(t)}$ and $q_{E(t)}$ as time-specific qualities in this explanation. Intuitively, the numerator tracks the measure of unemployed type H workers, while the denominator tracks the total measure of unemployed workers. To see this, consider these terms pairwise. The left-most of these account for already-unemployed workers—the size of this pool is $1 - E_{ss}$, of which proportion $q_{U(t)}$ are type H at time t.

The right three terms correspond to flows. $\eta_{ss}dt$ workers are removed instantaneously from the pool by hiring, and because firms do this selectively, the proportion $q_H\left(q_{U(t)}\right) > q_{U(t)}$ of these are type H. This reflects the previously-discussed fact that firms remove type H workers disproportionately, so hiring exerts downward pressure on the unemployment pool's quality.

In turn, quits draw randomly from employed workers, so high types make up $\pi dt E_{ss} q_{E(t)}$ of the $\pi dt E_{ss}$ job leavers at time t. There is no mechanism in this environment for unemployed workers to be negatively selected into employment, so $q_{E(t)}$ must be greater than $q_{U(t)}$ in equilibrium. Quits therefore push the unemployment pool quality upward.

Note that workers are fired after duration t_{ss}^* of unproductive employment, so the firing intensity at time t depends on two things: (1) the intensity and quality of hirings at time $t - t_{ss}^*$ and (2) how many of these workers neither quit nor reveal themselves to be type H before t. Obviously, type L workers will never generate output, so the proportion $e^{-\pi t_{ss}^*}$ of these workers (those who do not quit) will be fired at time t. In contrast, the proportion $e^{-(\lambda+\pi)t_{ss}^*} < e^{-\pi t_{ss}^*}$ of type H workers will reach this threshold, so low types will be disproportionately represented among those fired.

Thus, the steady-state equilibrium requires a precise balance between hirings, quits, and firings. In the absence of economic volatility, selective hiring ensures that this balance is stable. The next section, however, will show how even small shocks can disrupt this stability.

5. Shocks and Employment Dynamics

In this section, we will explore the dynamics of employment in response to economic shocks. Specifically, we will consider negative shocks to Y, which we will analyze in two cases.[31] To provide clear and simple intuition for labor market dynamics, we will first consider an unanticipated permanent shock. We will then analyze dynamics when this shock is known to be transitory; the implications for jobless recoveries will be considered in this context.

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[31] In a broader economic context, this may reflect a reduction in aggregate demand. Alternatively, to avoid focusing on the causes of the change in Y, we could interpret this as a generic decline in productivity.

5.1. Simplest Case: Permanent Shock

Suppose that, at time $t = \hat{t}$, the payoff Y falls unexpectedly from Y_{ss} to $Y_{ss} - z$. Suppose also that, before this shock, the economy was settled at its steady state employment equilibrium. The initial impact of this shock on firm behavior occurs through the threshold rule p^*, which rises from

$$p^*_{ss} = \frac{\widetilde{w}(r + \pi)}{\lambda [Y (r + \lambda + \pi) - \widetilde{w}]} \text{ to } p^*_{\hat{t}} = \frac{\widetilde{w}(r + \pi)}{\lambda [[Y - z](r + \lambda + \pi) - \widetilde{w}]} > p^*_{ss}$$

The decrease in Y makes firms less patient in learning about worker types—the payoff associated wth type H workers has decreased (and also decreased relative to the payoff associated with type L workers, which is 0), so the value of learning about worker type has also decreased. Hence, the threshold belief for terminating workers rises.

For notational purposes, let us define t_z to be the new time after hiring associated with the economic shock. This t_z applies to firms who hired workers before the shock occurred. Such firms will now be willing to wait t_z after their initial hires without a payoff before cutting ties— if they have already waited for some time t in the range $[t_z, t^*_{ss}]$, they will fire their workers immediately. As with t^*_{ss} in the previous section, we can express this t_z in terms of $p^*_{\hat{t}}$ and $q_{U(ss)}$:

$$t_z = \frac{1}{\lambda} \ln \left[\left(\frac{1 - p^*_{\hat{t}}}{p^*_{\hat{t}}} \right) \left(\frac{q_{U(ss)}}{\alpha (1 - q_{U(ss)})} \right) \right]$$

With this notation established, we can begin to characterize the impact of this shock on the economy. The mass of firms who fire their workers immediately after the shock is visually depicted in Figure 7 (below).
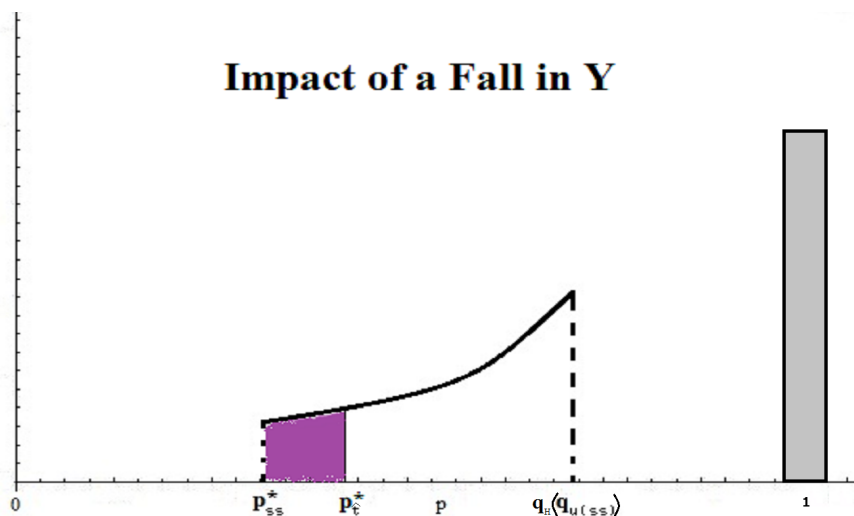

Figure 7: Firings after the shock

These workers fired in response to the shock (in the belief region $\left[p_{ss}^*, p_{\hat{t}}^*\right]$ are of better average quality than those fired in the preexisting steady-state. Note that this is perfectly in line with the standard intuition promoted in Nakamura (2008) and Lockwood (1991), among others. The standards for termination rise, so the quality of those terminated rises as well–this remains true in this model. The departure from this standard result is based not only on fired workers, but also on the mixture of fired workers and those who quit. This will be developed later in this section, when we characterize the evolving quality of the unemployment pool (see Proposition 4).

Next, consider the rise in the unemployed population—the unemployment pool will expand to include these newly fired workers. Formally, the measure of unemployed workers will rise from $1 - E_{ss}$ to

$$1 - E_{ss} + \frac{\eta_{ss}}{dt} \int_{t_z}^{t_{ss}^*} \left[q_H \left(q_{U(ss)} \right) e^{-(\pi + \lambda)s} + \left(1 - q_H \left(q_{U(ss)} \right) \right) e^{-\pi s} \right] ds$$

where the second component in this expression describes the mass of workers fired in immediate response to the shock. To see the intuition for this, consider the path of beliefs followed by a firm hiring a worker at time 0 in the steady state, where the firm neither realizes output nor has its worker quit before time t_{ss}^*. Of the workers hired in the steady state, proportion $q_H \left(q_{U(ss)} \right)$ of these are type H. Because firms with type H workers can leave this belief path either through a worker quitting or through a realization of output, proportion $e^{-(\pi + \lambda)t}$ of the firms hiring type H workers remain on this belief path after time t. In turn, firms with type L workers can leave this path only if this worker quits, so proportion $e^{-\pi t}$ of these firms remain on the path at time t. Thus, this expression corresponds precisely to the mass of new fires depicted in Figure 7.

Of course, there is information in these firing decisions–the firms who fire workers in response to the shock have beliefs in the range $\left[p_{ss}^*, p_{\hat{t}}^*\right]$. While these newly fired workers are better on average than those fired in the steady-state (at belief p_{ss}^*), they still represent the lowest belief range of previously employed workers. As such, relative to the set of employed workers, this group has disproportionately few type H workers. Yet, this group's quality alone is insufficient to lower the unemployment pool quality—the size of this group also plays a crucial role in the evolution of the unemployment pool.

In the steady-state, there was a precise balance in the flow to unemployment between quitting workers (of which proportion $q_{E(ss)} > Q$ were type H) and fired workers (of which proportion p_{ss}^* were type H)—this balance helped maintain the quality of the unemployment pool. After the shock, the negative pressure on unemployment pool quality from the mass of directed firings overwhelms the positive pressure from the flow of quitting workers (which is always of order dt). Thus, even though the workers fired in response to this shock are better (on average) than those fired in the steady-state, the unemployment pool decreases in quality.

Proposition 4: $\exists \overline{z} > 0$ *such that for* $z \in (0, \overline{z})$, *the proportion of type H workers in the unemployment pool immediately following the shock* $(Y \longrightarrow Y - z)$ *falls to*

$$q_{U(\hat{t})} = \frac{(1 - E_{ss}) q_{U(ss)} + \frac{\eta_{ss}}{dt} \int\limits_{t_z}^{t^*_{ss}} q_H \left(q_{U(ss)} \right) e^{-(\pi + \lambda)s} ds}{1 - E_{ss} + \frac{\eta_{ss}}{dt} \int\limits_{t_z}^{t^*_{ss}} \left[q_H \left(q_{U(ss)} \right) e^{-(\pi + \lambda)s} + \left(1 - q_H \left(q_{U(ss)} \right) \right) e^{-\pi s} \right] ds} < q_{U(ss)}$$

(See Appendix A for proof)

To see why we must bound values of z above for this to hold, consider the extreme case of $z > Y - \frac{w}{\lambda}$. Such a shock would be so large that even firms certain about having a type H worker would not continue to employ them, so we would have full unemployment, and $q_{U(\hat{t})}$ would rise to $Q > q_{U(ss)}$. Clearly then, for z sufficiently close to this range, so many workers will enter unemployment that the unemployment pool quality will rise. We can sensibly ignore such cases as disconnected from reasonable applications of this analysis.

Of course, this contamination of the unemployment pool is not the only response to this shock. As in the previous section, let q^*_U represent the unemployment pool quality at which firms are indifferent between hiring and not. For our purposes here, we write this as a function of Y: $q^*_U \equiv q^*_U(Y)$ (so $q^*_U(Y)$ reflects firm hiring standards at the payoff level Y). Then we can show that hiring standards rise along with firing standards.

Lemma 2: $q^*_U(Y)$ *is strictly decreasing in* Y.

(See Appendix A for proof)

Since the economy was at its steady-state before the shock (and since there was free entry in this steady state), the value of hiring a worker from the pool of unemployed at Y_{ss} was 0 (meaning $V\left(q_{U(ss)}\right) = c$). Obviously, conditional on employing a worker with belief $p_t > p^*$, the firm's value is monotonically increasing in Y. Thus, with Y now at $Y - z$, $V\left(q_H \left(q_{U(ss)} \right)\right) < c$ and firms will require a higher initial belief $q_H \left(q^*_U(Y - z) \right)$ (and, in turn, a higher unemployment pool quality) to justify hiring a worker.

As we have seen, though, this increased standard is compounded by a drop in the unemployment pool quality—the rising standards for hiring and firing have forced a wedge between actual market conditions and those necessary for sustained hiring. As a result, hiring will cease completely. Hiring will resume only when the unemployment pool quality has recovered sufficiently to collapse this wedge—voluntary employee quits are the channel through which this will occur. Only after enough remaining employees have voluntarily entered the unemployment pool will hiring resume. Assuming that employee quits continue at a constant rate following the shock, we can analytically characterize the duration without hiring:

Proposition 5: *After the output shock* $Y \longrightarrow Y - z$, *hiring will cease for the duration* $\widehat{t_H} > 0$ *(an expression for* $\widehat{t_H}$ *appears in the appendix). If* $q_U^* (Y - z) < Q$, $\widehat{t_H}$ *is finite and satisfies* $q_U^* (Y - z) = f_1 \left(\widehat{t_H} \middle| t_{ss}^*, t_z \right)$ *(an expression for* $f_1 \left(\widehat{t_H} \middle| t_{ss}^*, t_z \right)$ *appears in the formulation of this proposition given in Appendix A).*

(See Appendix A for proof)

First, note that, because there can be no positive selection into unemployment, $q_{U(t)} < Q$, $\forall t$. Thus, if z is so large that $q_U^* (Y - z) \geq Q$, then it is impossible for the unemployment pool quality to reach a level at which hiring can resume, and employment will converge to 0 over time.[32]

Regarding the case in which $q_U^* (Y - z) < Q$, the interested reader will see that the expression provided in Appendix A for $f_1 \left(\widehat{t_H} \middle| t_{ss}^*, t_z \right)$ is long and messy. Despite this, the intuition behind the condition $q_U^* (Y - z) = f_1 \left(\widehat{t_H} \middle| t_{ss}^*, t_z \right)$ reduces to an accounting exercise. $f_1 \left(\widehat{t_H} \middle| t_{ss}^*, t_z \right)$ represents the proportion of type H workers in the unemployment pool when time $\widehat{t_H}$ has elapsed after the shock. To express this, we must account for workers who:

(i) were already unemployed at $\widehat{t}$

(ii) were fired immediately after the shock

(iii) were fired upon beliefs reaching the new threshold $p_{\widehat{t}}^*$ at some time $t \in \left[\widehat{t}, \widehat{t} + \widehat{t_H} \right]$

(iv) quit their jobs at some time $t \in \left[\widehat{t}, \widehat{t} + \widehat{t_H} \right]$ while at firms with beliefs in the range $p_t \in \left[q_H \left(q_{U(ss)} \right), p_{\widehat{t}}^* \right]$

(v) revealed themselves to be type H workers before the shock and quit their jobs at some time $t \in \left[\widehat{t}, \widehat{t} + \widehat{t_H} \right]$

(vi) revealed themselves to be type H workers after the shock and quit their jobs at some time $t \in \left[\widehat{t}, \widehat{t} + \widehat{t_H} \right]$

These six groups are represented (in order) in both the numerator and denominator in the expression for $f_1 \left(\widehat{t_H} \middle| t_{ss}^*, t_z \right)$ given in Appendix A (in the numerator, only the type H workers are counted). To illustrate this further, the evolution of the employment level and the unemployment pool quality after the shock are depicted below.

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[32] In the case where $q_U^* (Y - z) = Q$, $q_{U(t)}$ will converge to Q as $t \longrightarrow \infty$, but it will not reach Q in finite time.
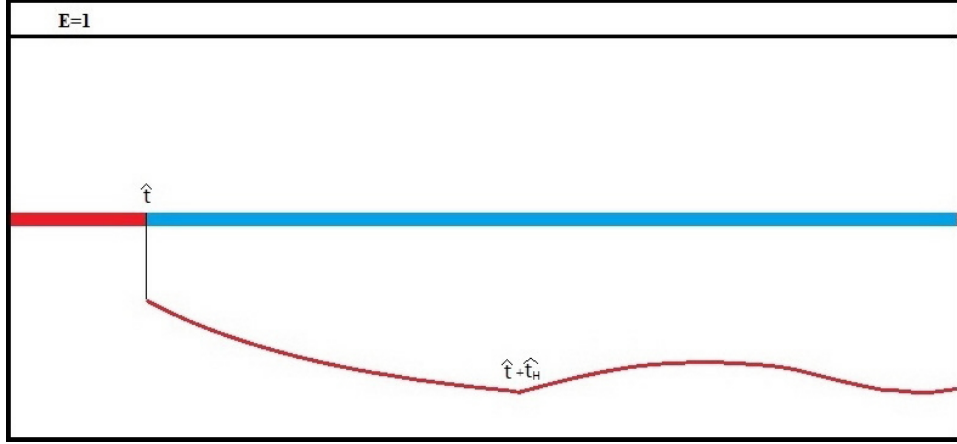
30

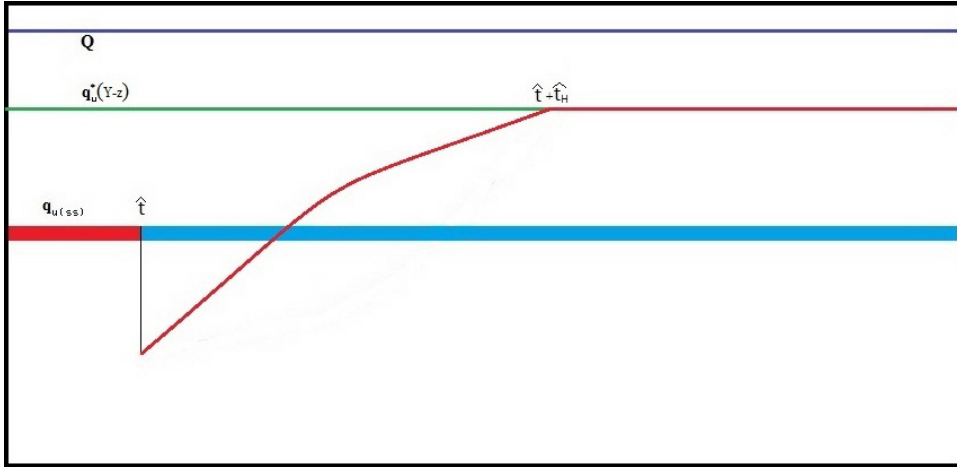**Figure 8:** Evolution of employment level after shock

Figure 9: Evolution of unemployment pool quality after shock

It is crucial to note that the duration obtained in Proposition 5 and depicted above in Figures 8 and 9 is computed assuming that workers continue to quit jobs voluntarily after the shock at the same rate as they did in the previous steady-state. In reality, voluntary quits plummet during a recession (as shown in Figure 1). If this were to happen in the model, the duration without hiring would be magnified drastically. As voluntary quits are the main source of upward pressure on the unemployment pool quality, reducing the volume of these quits would slow the recovery in quality of the unemployment pool. As such, we can view this duration $\widehat{t_H}$ as a lower bound on the duration for which hiring should cease.

Figures 8 and 9 make clear that the unemployment pool will grow after the shock—unmitigated by hiring—through the six channels listed above until the quality of this pool has risen to the new threshold required for hiring to resume. It is easy to see that the new, lower output equilibrium will have more unemployment and a higher unemployment pool quality. This is in line with the standard view about the composition of the unemployment pool in recession—more people are unemployed, and hiring/firing standards are higher, so the pool must be better. Of course,

this model highlights the flaws with this logic: First, the transition from the first equilibrium to the second involves a significant period with a lower quality unemployment pool. Second, the second equilibrium may not be a permanent state of the economy. A recession is commonly viewed as a temporary productivity shock—if productivity rebounds during the transitional period, this equilibrium with a better unemployment pool may never be reached in the first place. We address this issue below.

5.2. Transitory Shocks and Jobless Recoveries

To better understand the dynamics induced by a temporary economic downturn, consider one further variation on this economic shock structure. Suppose that, as before, there is an unanticipated output shock $Y \longrightarrow Y - z$ at time $\widehat{t}$, and that firms respond to this immediately. Rather than permanently remaining at this level, however, suppose that output will permanently return from $Y - z$ to Y at some point in the future, and suppose that firms know this. In particular, suppose that the "recovery date" follows a Poisson distribution with parameter γ, so that at each point in time during the "recession," the economy recovers with probability γdt. Then we can show that, for a range of recovery dates $t \in \left[\widehat{t}, \widehat{t} + \widehat{t_{Y-z,Y}}\right)$, there will be no hiring for some time *even after the economy has recovered to the previous level.*[33]

Toward formalizing this result, define $p^* (Y - z, Y)$ to be the termination belief level after the shock but before the recovery, and define $t^* (Y - z, Y)$ to be the associated time firms will wait without output before terminating a worker. Further, define $q_U^* (Y - z, Y)$ to be the corresponding hiring threshold during this "recession," and again define $q_U^* (Y)$ to be the hiring threshold after the recovery. We can then establish the following:

Proposition 6: *Consider an unanticipated transitory output shock $Y \longrightarrow Y - z$ to the steady-state at time $\widehat{t}$, immediately after which it is known that output will rebound $Y - z \longrightarrow Y$ at Poisson-distributed times with parameter γ. After firms respond optimally to this shock at $\widehat{t}$, if the recovery occurs before time $\widehat{t} + \widehat{t_{Y-z,Y}}$ (where $\widehat{t_{Y-z,Y}}$ satisfies $q_U^* (Y) = f_2 \left(\widehat{t_{Y-z,Y}} \,\middle|\, t_{ss}^*, t_{Y-z}^*\right)$, and an expression for $f_2 \left(\widehat{t_{Y-z,Y}} \,\middle|\, t_{ss}^*, t_{Y-z}^*\right)$ is provided in the appendix), **then the economy will remain without hiring for a positive duration of time even after the recovery**. Further, $\exists \overline{z} > 0$ such that for $z \in (0, \overline{z})$, both the likelihood and expected duration of a jobless recovery are increasing in the magnitude of the shock z.*

(See Appendix A for proof)

Because the unemployment pool has been contaminated by the equilibrium response to the initial shock, the labor market may be unable to sustain hiring *even after the recovery.* In

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[33] An expression for determining the duration $\widehat{t_{Y-z,Y}}$ will be provided in the proof of Proposition 6 in Appendix A.

a sense, this result analyzes the impact of temporarily increased firing standards without the impact of increased hiring standards (as the hiring threshold returns to its steady-state level after the recovery).[34] After the recovery, hiring will return sooner in this case than in the scenario considered in Proposition 5, but the key insight is the fact that stagnant hiring can persist even after other economic indicators have rebounded. Further, as the result above indicates, larger shocks may make these jobless recoveries more likely and longer-lasting.

6. Discussion

6.1. Robustness of Section 5 Results

"Weakening" the Results

Propositions 5 and 6 show that negative economic shocks can stop hiring completely both during and after recessions. In reality, hiring does not halt completely at such times; it merely slows (although it can slow significantly). The model presented is not intended to be quantitatively precise,[35] but the severity of the above results should provide *more* reason, not less, to take the analysis seriously. If these forces can weaken employment so greatly in the model, then even a small analog of this mechanism in reality may play a large role in the interplay between economic fluctuations and labor market dynamics.

Alternate Structure of Shocks

The previous section represented a recession as an unanticipated, discrete, one-time shock to productivity (which can be either permanent or transitory). This structure is used for intuitive and analytic simplicity, but the results can be shown to hold when shocks take different forms. Firstly, the shocks need not be discrete—continuous declines in productivity, in which Y falls proportionally to dt at each time increment dt, are sufficient to stop hiring in this model.

In addition, the shocks need not be exogenous, unanticipated changes in the model. At some cost of analytical tractibility, this labor market can be modeled in an environment where productivity varies according to general Markov transitions. The model's dynamic predictions apply to this setting as well.

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[34] In Appendix A, I also include an alternate formulation of this result (called Proposition 6.A) in which the initial unanticipated output shock $Y \longrightarrow Y - z$ at time $\widehat{t}$ is followed almost immediately by another unanticipated output shock $Y - z \longrightarrow Y$ at time $\widehat{t} + dt$, where firms have already responded to the first shock before the second occurs. Of course, a setting with two unanticipated shocks that are both expected to be permanent is farther from reality than the environment of Proposition 6, but it yields a similar result with simple, clear intuition.

[35] At the cost of analytical tractability, we could generate a decrease (rather than a halt) in hiring by introducing firm heterogeneity. This complication adds little to the current analysis, so it is omitted.

Match-Specific Productivity

A potential criticism of the model concerns the productivity of type L workers. Because employment imposes a flow cost of at least $\widetilde{w}$ on the firm, it is inefficient for type L workers to be employed. As discussed in Section 3, $\widetilde{w}$ need not represent the cost of effort, so this inefficiency may not be the fault of type L workers. Independently of the interpretation of $\widetilde{w}$, however, the results of Section 5 will persist even if match-specific productivity is incorporated and type L workers can be profitably employed.

Suppose the present model is modified so that type H workers are productive at a proportion μ_H of firms, while type L workers are productive at a proportion $\mu_L < \mu_H$ of firms (choosing $\mu_H = 1$ and $\mu_L = 0$ yields our original model). If individual worker productivity is i.i.d. across firms, the previous results will extend to this setting. Yet, type L workers are no longer inherently unemployable. If firms can observe not only workers' types, but also their firm-specific productivities, then all workers can be profitably employed.

More General Structures for Information and Learning

The employment dynamics predicted in Section 5 will be substantively unchanged in variety of alternative informational structures. For the present study, I have assumed that information arrives in the form of positive, perfectly informative (perfect good news) signals.[36] The analysis will be extremely similar if these signals are imperfect.[37] Departing from Poisson learning, the results will extend to the case where firms learn about worker output based on Brownian motion with unknown drift $\mu_\theta \in \{\mu_H, \mu_L\}$ (as in Bolton and Harris, 1999). Predictions will also be similar if instead μ_θ can take on a continuum of values and firm priors are normally distributed (as in Jovanovic, 1979).[38] In each of these cases, firms fire their least productive workers. Firing standards rise during recessions, and the resulting increase in firing causes a decline in unemployment pool quality.

6.2. Dynamic Equilibria: Aggregate Fluctuations without Shocks

The forces that stabilize the employment level—discussed in Section 4—will help preserve equilibrium even if the labor market is not at a steady-state. Consider selective hiring, which lowers

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[36] In the spirit of Milgrom (1981) and the information economics literature more generally, consider a signal to be "good news" if it indicates high quality and "bad news" if it indicates low quality. The good or bad news is "perfect" if it fully reveals quality and "imperfect" otherwise.

[37] These results may be weakened under bad news learning. If the recession-induced shock is small, firms will fire their workers only if they have beliefs sufficiently close to the steady-state termination threshold. With imperfect bad news signals, downward updating is discrete, so it is possible that no firms will have beliefs in this range. Under perfect bad news learning, each signal will result in termination, so no workers will be employed below the initial post-hiring belief.

[38] In this setting, the termination threshold will depend on employment duration.

the unemployment pool's quality. Firms will refuse to hire if this quality is too low, so free entry dictates how much hiring the labor market can sustain. In a steady-state, hiring is fixed at an intensity to preserve the unemployment pool's quality, but free entry and hiring can also be used to sustain equilibria in which employment varies. In these cases, the aggregate intensity of hiring adjusts to balance its own effects on the unemployment pool with those of firings and quits. When firings intensify, hirings must decrease to prevent a fall in quality; conversely, when firings slow, hirings must increase to prevent a rise in quality.

This dynamic rebalancing can generate lasting fluctuations. Consider a simple example— suppose that the intensity of firing is high at time t, which implies that the time t hiring intensity must be low. Recall from Section 4 that, because of the learning process, the firing intensity at time t depends on the hiring intensity at $t - t^*$ (when the fired workers were hired). At time $t + t^*$, then, there will be limited firing and much hiring, and the pattern will continue. Thus, the labor market can sustain equilibria with evolving employment levels; some of these equilibria are cyclical. Additionally, output itself will vary, rising and falling with employment. I discuss this in more detail in Appendix B.

The permanence of this cyclical pattern is a consequence of the Poisson learning structure and its fixed time-to-firing. Other settings might rule out these perpetual cycles, but the short-term effects of hiring and firing fluctuations would persist under more general information structures.[39] In fact, the intuition for permanent cycles extends not only to these general short-term effects, but also to actual labor market applications.

To see this, consider a labor market in which firms learn about worker productivity during employment. In this labor market, suppose that there are no variations in demand or worker productivity, but that the aggregate intensities of hiring and firing can vary. Take these variations as given. In periods with more aggregate hiring, firms accumulate new workers, and these workers have more uncertain productivity. Not all of this uncertainty will be resolved at a single moment in the future (as in the good news learning structure modeled here), but it will all be resolved eventually. Thus, it is reasonable that aggregate hiring increases can lead to future firing increases. In turn, periods with more aggregate firing involve the (negative) resolution of uncertainty. Such periods will generate a lower quality, more negatively-selected unemployment pool. Firms then face lower expected returns to hiring workers, so hiring slows.

As this application illustrates, these dynamic equilibria provide two insights beyond the recessions mechanism highlighted throughout the paper:

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[39]If learning were based on Brownian motion, for instance, firm firing threshold beliefs would be reached at a continuous distribution of employment durations (in contrast to the pre-firing delay t^* we saw here). As a result, the impact of a hiring intensity change on firings would not occur at a fixed future time, but it would instead be spread over a distribution of future times. This effect would therefore be muted at each time in the distribution. Fluctuations would dissipate over time, but not immediately. Though smaller, short-term effects would remain.

(1) Independently of the state of the economy, there must be an aggregate balance in firm restructuring decisions that impact the labor market. This market can support only so much combined hiring and firing at once. For example, a sector in which many firms are firing unproductive workers may experience a lull in hiring due exclusively to the changing composition of the unemployment pool. This suggests that there can be aggregate fluctuations in employment and output *even in the absence of economic shocks* (to demand, to production costs, etc.).

(2) Further, fluctuations in hiring and firing can contribute to future fluctuations. Because these fluctuations change the aggregate productive uncertainty that firms face, their most significant consequences may be realized well-beyond the immediate future. Thus, serial correlation may be unable to characterize the dependence of these fluctuations across time.

6.3. Extensions

For theoretical applications beyond the scope of this paper, the framework developed here should be viewed merely as a starting point for modeling labor markets. The model can be enriched in a number of ways while remaining analytically tractible. I discuss two simple examples below.

Endogenous Quitting

The simplifying assumption of a constant quit rate π is standard in many models of labor markets. In addition, though, I use this structure because it weakens my results. If worker quit rates varied in the model as they do in the data, far *fewer* workers would quit during recessions. The unemployment pool's quality would fall even more than the model predicts, and the duration of jobless recoveries would be magnified. In this sense, the constant quit rate assumption demonstrates the robustness of the model's predictions.

Of course, we may want to use this framework more generally to study how learning and private information impact the labor market equilibrium. In this case, it may be useful to endogenize quits. This can be done in many ways; a simple extension to the current model would have workers receive (still at intensity π) a stochastic cost, x, of continued employment at the current firm. If x is always infinite, this is equivalent to the current model. More generally, x could be distributed according to the c.d.f. $F(x)$ over $\mathbb{R}$. We might even want this cost to correlate with worker types, so that job leavers are selected (positively or negatively) on productivity.

For simplicity, suppose these costs are observed jointly by the worker and firm. Then costs greater than the firm's remaining surplus would result in the worker quitting. In turn, realized

costs smaller than this surplus would prompt wage renegotiation.[40] Additionally, such wage renegotiation would change the firm's firing standard. Workers with higher wages would face more frequent dismissals; Schmieder and von Wachter (2010) document this pattern precisely.

To take quitting seriously, we would obviously require a more developed and better-justified theory of job leaving decisions.[41] Such a theory could be incredibly powerful when embedded in the dynamic equilibrium framework developed in this paper. The example above demonstrates this power—this simple addition to the model impacts both the evolution of wages and the relationship between these wages and job turnover. Incorporating a more thorough treatment of quitting could yield a variety of more nuanced, empirically-testable predictions.

Poaching and Job-to-Job Transitions

This paper's analysis has focused on unemployment, so we have assumed throughout that firms can hire only from the unemployment pool. This may be a reasonable approximation for the low-wage, low-human capital jobs that are responsible for much of the recent surge in unemployment. However, we can incorporate job-to-job transitions in this model and demonstrate how these change equilibrium labor market conditions.

Consider a simple modification to the model allowing firms to hire currently employed workers at cost c_E. In attempting to hire these workers, firms cannot observe the worker's output at her current job—this remains the private information of her current employer. The "poaching" firm can, however, view the worker's duration of employment at her current firm, and all firms are identical, so equilibrium hiring/firing behavior is commonly known. In a steady-state equilibrium, tenure at the current firm will be informative about worker types.

Only type H workers will be retained after a duration t_{ss}^* of employment, so hires from this group are preferred to those from the unemployment pool. The current employer must therefore raise wages to defend these workers from the bidding of outside firms. Free entry drives the offers of these firms, so they will offer wages at which they expect value 0 from the type H workers. Further, only outside firms face the poaching cost c_E, so the current employer will retain precisely this expected value after raising wages to defend this worker from other firms. The current employer can always defend successfully, and no poaching will occur in equilibrium. We can use this setting to explore this "testing period" and its impact on wage dynamics.

Also note that this pooled-hiring equilibrium cannot exist unless each worker cannot retain private information about her type when reentering the unemployment pool. To see this, consider

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[40]Wage decreases can occur if x is allowed to take on negative values.

[41]An interesting case is that in which only the worker can observe this cost. The worker would have an incentive to overreport this cost, but this incentive would be tempered by the reduced time to firing that accompanies higher wages. If some overreporting occurs in equilibrium, it will come disproportionately from type H workers, who are more confident that they will be retained at the firing threshold.

two unemployed workers—one knows she is type H and another knows she is type L. The type H worker would be willing to accept a lower initial wage because of the higher wages she might get by surviving through duration t_{ss}^*. The type L worker expects to be fired at time t_{ss}^* and is thus unwilling to accept this lower wage. The employer may therefore be able to separate worker types even without being able to commit to an incentive contract—implicit incentives are provided by the wage response to poaching.

7. Conclusions

In this study, I have considered a new mechanism, in which changes in the quality of those entering unemployment can generate both a long post-recession period with limited hiring and large numbers of individuals reaching long-term unemployment. If employers have private information about worker ability, then periods in which many firms make firing decisions will involve many low-ability workers entering the unemployment pool. Of course, previous research argued that these low-ability workers should still be better than those fired at other times (because firing standards are higher during recessions). What these studies have overlooked is the other impact of increasing firing standards—a significant increase in the number of workers fired. Workers fired before the recession were balanced in the unemployment pool by workers voluntarily quitting jobs, and these quitting workers need not have disproportionately low ability. During the recession, this balance was lost, and the unemployment pool may have worsened as a result. If true, this would at least partly explain the continued hesitancy of firms to hire, and this could grease the path to long-term unemployment for these disproportionately low-quality workers who would struggle to find work regardless.

To assess the impact of this compositional change on employment, I have provided an empirical strategy for detecting the role of changing flows to unemployment in firm hiring decisions. I have implemented this using CPS monthly employment data, and I have provided evidence linking these compositional changes to hiring decisions. The empirical patterns provided cannot be explained by human capital depreciation, negative selection of LTU, or other common explanations of persistent long-term unemployment, so this constitutes evidence that the compositional change mechanism impacts hiring independently of these other factors. As such, more investigation is warranted regarding the role of these changing flows on employment dynamics.

Building on this empirical motivation, I have formalized this mechanism in a dynamic framework that integrates employer learning and private information into a labor market equilibrium. The framework itself is a tool that merits further development and application. As I have demonstrated the danger of drawing conclusions about dynamic environments from comparisons of static models, the availability of a tractable dynamic framework for analysis of the labor

market is an opportunity to investigate whether other analyses of this setting have been flawed.

In my present analysis, I show that an economic shock which raises firing standards can not only generate significant unemployment, but also discourage firms from hiring for a sustained period of time. The conditions amplify each other, and this will be worsened significantly if the stagnant labor market discourages workers who want to quit their jobs from doing so. (In generating the results, I have assumed that quits will continue regardless of labor market conditions, so I may be understating the potential impact of this mechanism). The severity of the results suggests that, even if this mechanism plays a small role in actual labor markets, its impact may be large during economic downturns.

In addition to this, I show that the stagnant labor market can persist even after the economy has otherwise recovered from the shock, so this may offer insight regarding the "jobless recoveries" that have followed the past several recessions. Further, I show that the model does not even require a shock to generate employment/output fluctuations. In hiring and firing, firms worsen the labor unemployment pool faced by other firms, and an increase in firing decreases the amount of hiring the economy cannot support. Hiring and firing have opposing effects on aggregate output, so even in the absence of external shocks to the labor market, variation in the relative intensities of these flows can generate volatility throughout the economy.

These empirical and theoretical results offer strong motivation for future work investigating the role of this mechanism in recessions and in labor markets more generally.

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APPENDIX A: Proofs of Theoretical Results

Lemma 1: *In any nontrivial steady-state employment equilibrium, $w = \widetilde{w}$.*

Proof: *Obviously, we need only consider cases in which $E_{ss} \in (0,1)$. Clearly, no one would work at lower wages, so $w \geq \widetilde{w}$. Suppose, then, that firms offer equilibrium wages $w > \widetilde{w}$. There is positive unemployment, so the unemployment pool is of sufficiently low quality that firms are unwilling to hire more at the wage w. These unemployed workers, however, could benefit by undercutting the market wage and accepting some offer $w' \in (\widetilde{w}, w)$, so this cannot be an equilibrium, and we have the result.* ∎

In addition, we can show that the above result implies a constant hiring intensity and constant qualities of the employed and unemployed.

Corollary: *In any nontrivial steady-state employment equilibrium, $\eta_t = \eta_{ss}$, $q_{E(t)} = q_{E(ss)}$, and $q_{U(t)} = q_{U(ss)}$, $\forall t$.*

Proof: *Because $E_{ss} \in (0,1)$, free entry implies that $V\left(q_H\left(q_{U(t)}\right)\right) \leq c$ at the wage $\widetilde{w}$. For $\pi > 0$ (assumed), there must be hiring in equilibrium—otherwise the employment level would decline due to the flow of quitting workers. Given that there is hiring, we can establish $V\left(q_H\left(q_{U(t)}\right)\right) \geq c$ also, so $V\left(q_H\left(q_{U(t)}\right)\right) = c$. Since $w = \widetilde{w}$ and $V\left(q_H\left(q_{U(t)}\right)\right) = c$, we know that $q_{U(t)}$ must be fixed at $q_{U(ss)}$, $\forall t$. In turn, the accounting identity $Q = E_{ss}q_{E(t)} + (1 - E_{ss})q_{U(ss)}$ tells us that $q_{E(t)} = q_{E(ss)}$ must also be fixed.*

Since w is fixed, $p^*_{ss} = \frac{\widetilde{w}(r+\pi)}{\lambda[Y(r+\pi+\lambda)-\widetilde{w}]}$ must also be fixed. The constant values of p^*_{ss} and $q_{U(ss)}$ together require $t^*_{ss} = \frac{1}{\lambda} \ln\left[\left(\frac{1-p^*_{ss}}{p^*_{ss}}\right)\left(\frac{q_{U(ss)}}{\alpha\left(1-q_{U(ss)}\right)}\right)\right]$ to be fixed as well. By solving for the hiring intensity to equate inflows and outflows to unemployment (or simply of type H workers), we verify that constant values of E_{ss}, $q_{U(ss)}$, and t^*_{ss} imply a constant hiring intensity η_{ss}. ∎

We now proceed with the existence/uniqueness result. First, define $\bar{c} = V\left(\frac{Q}{Q+(1-Q)\alpha}\right)$ to be the maximum hiring cost at which there can be any employment in equilibrium. (Selection into employment must always be positive, so we must always have $q_{U(ss)} < Q$. With zero employment, $q_{U(ss)} = Q$.)

Proposition 1: *For any combination of parameters $\{Y, \widetilde{w}, \lambda, \pi, r, Q, \alpha\}$, $\exists \underline{c} \in (0, \bar{c})$ such that for hiring costs $c \in (\underline{c}, \bar{c})$, there exists a unique nontrivial steady-state employment equilibrium.*

Proof: *Consider first the conditions this equilibrium must satisfy. Firm optimality (value-matching, smooth-pasting) provides us with our termination belief $p^*_{ss} = \frac{\widetilde{w}(r+\pi)}{\lambda[Y(r+\pi+\lambda)-\widetilde{w}]}$ and the firm value function. Free entry must hold with equality, so the combination of free entry and the firm optimality conditions pin down the firm belief at which it can recover exactly the hiring cost c. Let us define q^*_H to be this belief, so this must satisfy $V\left(q^*_H\right) = c$ and $q^*_H = \frac{q_{U(ss)}}{q_{U(ss)}+\alpha\left(1-q_{U(ss)}\right)}$.*

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Further, the constant employment level (along with constant qualities in employment and unemployment) means that the steady-state hiring intensity must satisfy two conditions: (1) inflows to and outflows from unemployment must be equal and (2) type H workers among inflows to and outflows from unemployment must be equal. Condition (1) requires

$$\eta_{ss}dt = \eta_{ss}dt \left[q_H^* e^{-(\pi+\lambda)t_{ss}^*} + (1 - q_H^*) e^{-\pi t_{ss}^*} \right] + \pi E_{ss} dt$$

while condition (2) requires

$$\eta_{ss} q_H^* dt = \eta_{ss} q_H^* dt e^{-(\pi+\lambda)t_{ss}^*} + \pi E_{ss} q_{E(ss)} dt$$

By solving each of these equations for η_{ss}, equating them, and by using the labor force condition that $Q = E_{ss}q_{E(ss)} + (1 - E_{ss}) q_{U(ss)}$ to substitute for $q_{E(ss)} = \frac{Q - (1 - E_{ss})q_{U(ss)}}{E_{ss}}$, we can obtain an expression for E_{ss} in terms of Q, q_H^*, α, π, λ, and t_{ss}^*:

$$E_{ss} = \left(\frac{Q - q_H^* [\alpha + Q(1 - \alpha)]}{1 - q_H^*} \right) \left(\frac{1 - e^{-\pi t_{ss}^*} + q_H^* e^{-\pi t_{ss}^*} \left(1 - e^{-\lambda t_{ss}^*} \right)}{1 - \alpha + e^{-\pi t_{ss}^*} [\alpha - e^{-\lambda t_{ss}^*}]} \right)$$

This equation has used all of the equilibrium conditions, and if this expression yields an employment level $E_{ss} \in (0, 1)$, we have established existence. Toward this end, note that substituting our upper bound on the equilibrium quality of new hires, $\frac{Q}{Q+(1-Q)\alpha}$, for q_H^* in the expression for E_{ss} above yields an employment level of 0. (Recall that $\overline{c} = V\left(\frac{Q}{Q+(1-Q)\alpha} \right)$ is the upper bound on our desired range of hiring costs.) We thus need to show that E_{ss} is decreasing in c at $c = \overline{c}$ (so that $E_{ss} \in (0,1)$ for c less than, but sufficiently close to $\overline{c}$).

In E_{ss} above, we can see that changes in c affect the equilibrium employment level through only q_H^* and $t_{ss}^* = \frac{1}{\lambda} \ln \left[\left(\frac{1-p_{ss}^*}{p_{ss}^*} \right) \left(\frac{q_H^*}{1-q_H^*} \right) \right]$. Because p_{ss}^* does not depend on c, substituting for t_{ss}^* will provide us with an expression for E_{ss} which c affects through only q_H^*. Making this substitution yields:

$$E_{ss} = \frac{q_H^* (Q - q_H^* [\alpha + Q(1 - \alpha)]) \left[(1 - p_{ss}^*)^{\frac{\pi+\lambda}{\lambda}} (q_H^*)^{\frac{\pi}{\lambda}} - (1 - q_H^*)^{\frac{\pi+\lambda}{\lambda}} (p_{ss}^*)^{\frac{\pi}{\lambda}} \right]}{(1 - q_H^*) \left[(1 - \alpha) \left[(1 - p_{ss}^*) q_H^* \right]^{\frac{\pi+\lambda}{\lambda}} + \left[p_{ss}^* (1 - q_H^*) \right]^{\frac{\pi}{\lambda}} \left(q_H^* [\alpha + p_{ss}^* (1 - \alpha)] - p_{ss}^* \right) \right]}$$

The firm value function $V(\cdot)$ is increasing in beliefs, so an increase in c must yield an increase in q_H^*. Also, $V(\cdot)$ is continuous in beliefs, so q_H^* depends continuously on c. Thus, establishing existence is reduced to verifying that this expression for E_{ss} is decreasing in q_H^*. A bit of tedious algebra can demonstrate that

$$\left. \frac{\partial E_{ss}}{\partial q_H^*} \right|_{q_H^* = \frac{Q}{Q+(1-Q)\alpha}} = \frac{-Q \left[(1 - p_{ss}^*)^{\frac{\pi+\lambda}{\lambda}} (q_H^*)^{\frac{\pi}{\lambda}} - (1 - q_H^*)^{\frac{\pi+\lambda}{\lambda}} (p_{ss}^*)^{\frac{\pi}{\lambda}} \right]}{(1 - q_H^*) \left[(1 - \alpha) \left[(1 - p_{ss}^*) q_H^* \right]^{\frac{\pi+\lambda}{\lambda}} + \left[p_{ss}^* (1 - q_H^*) \right]^{\frac{\pi}{\lambda}} \left(q_H^* [\alpha + p_{ss}^* (1 - \alpha)] - p_{ss}^* \right) \right]}$$

$$< 0$$

To check the inequality, notice that the denominator is positive. Next, note that $(1-p_{ss}^*)^{\frac{\pi+\lambda}{\lambda}}(q_H^*)^{\frac{\pi}{\lambda}} >$ $(1-q_H^*)^{\frac{\pi+\lambda}{\lambda}}(p_{ss}^*)^{\frac{\pi}{\lambda}}$ *whenever* $q_H^* > p_{ss}^*$. *Recall that we have assumed throughout the analysis that* $\widetilde{w} < \frac{\lambda Y(r+\pi+\lambda)Q}{(r+\pi+\lambda)Q+\alpha(r+\pi)(1-Q)}$ *(because otherwise no combination of parameters could allow employment greater than 0 in equilibrium), so indeed* $q_H^* > p_{ss}^*$ *must hold for* q_H^* *sufficiently close to* $\frac{Q}{Q+(1-Q)\alpha}$. *Thus, the numerator and the entire object are negative, so our equilibrium* E_{ss} *is decreasing in c, and a nontrivial steady-state employment equilibrium exists for hiring costs sufficiently close to* $\overline{c}$.

Uniqueness follows immediately; to see this, notice that inflows/outflows condition (1) above implies

$$\eta_{ss} = \frac{\pi E_{ss}}{1 - q_H^* e^{-(\pi+\lambda)t_{ss}^*} - (1-q_H^*)e^{-\pi t_{ss}^*}}$$

while condition (2) implies

$$\eta_{ss} = \frac{E_{ss} q_{E(ss)}\pi}{q_H^*\left[1 - e^{-(\lambda+\pi)t_{ss}^*}\right]}$$
$$= \frac{\pi\left[Q - (1-E_{ss})\left(\frac{\alpha q_H^*}{1-q_H^*(1-\alpha)}\right)\right]}{q_H^*\left[1 - e^{-(\lambda+\pi)t_{ss}^*}\right]}$$

All parameters but E_{ss} *are determined in these. In response to varying* E_{ss}, *these expressions for* η_{ss} *have different slopes and cross only once, so only one hiring intensity can satisfy both conditions. In turn, the equilibrium is unique (if these conditions for* η_{ss} *are satisfied at an employment level in the* $(0,1)$ *range).* ∎

Corollary: *For any combination of parameters* $\{Y, \widetilde{w}, \lambda, \pi, r, Q, \alpha\}$ *satisfying* **(A1)** *and* **(A2)**, $\exists \underline{c} \in (0, \overline{c})$ *such that for hiring costs* $c \in (\underline{c}, \overline{c})$, *there exists a unique nontrivial steady-state employment equilibrium.*

Proof: **(A2)** *simply ensures that* $\overline{c} > 0$, *and the result above holds regardless of whether or not* **(A1)** *is satisfied. We can thus take the same approach as in establishing Proposition 1 above to show that c just below* $\overline{c}$ *will yield a unique nontrivial steady-state employment equilibrium.* ∎

Proposition 2: *In a nontrivial steady-state employment equilibrium, the firm's value function can be written analytically as:*

$$V_{ss}(p_t) = \begin{cases} \left(\frac{1}{r+\pi}\right)\left[\lambda p_t Y - \widetilde{w} + \lambda\left(\frac{\widetilde{w}(1-p_t)}{r+\pi+\lambda}\right)^{\frac{r+\pi+\lambda}{\lambda}}\left(\frac{(\lambda Y - \widetilde{w})p_t}{r+\pi}\right)^{-\frac{r+\pi}{\lambda}}\right] & \text{for } p_t \in [p_{ss}^*, 1] \\ 0 \text{ for } p_t \leq p_{ss}^* \end{cases}$$

Further, this threshold level is given by $p_{ss}^* = \frac{\widetilde{w}(r+\pi)}{\lambda[Y(r+\lambda+\pi)-\widetilde{w}]}$

Proof: *The firm's optimal strategy will incorporate a threshold rule—it will be optimal to retain the worker until the belief* p_t *falls to some* p^* *at which the outside option offers value equal to that of the current match. At this point, the firm will choose this outside option, which can be*

written as $\max\left\{V\left(q_H\left(q_{U(ss)}\right)\right) - c, 0\right\}$. Free entry requires that $V\left(q_H\left(q_{U(ss)}\right)\right) - c < 0$, so this outside option must always be 0. Additionally, this implies that the threshold p^* will be fixed at its steady-state level, so we can denote this cutoff by p_{ss}^*.

We can use the fixed outside option to write the firm's value function. When the firm's belief is above p_{ss}^* (so that it chooses to remain with its current employee), its value function[42] can be written as

$$rV\left(p_t\right) = \left[\lambda p_t Y - \widetilde{w}\right] - \pi V\left(p_t\right) + \lambda p_t \left[V\left(1\right) - V\left(p_t\right)\right] - \lambda p_t \left(1 - p_t\right) V'\left(p_t\right)$$

where we have used the standard approximation that

$$V\left(p_t - \lambda p_t \left(1 - p_t\right) dt\right) \approx V\left(p_t\right) - \lambda p_t \left(1 - p_t\right) V'\left(p_t\right) dt$$

and canceled out higher order terms. Further, substituting for the value of employing a type H worker, $V\left(1\right) = \frac{Y\lambda - w}{r + \pi}$, yields the following first-order ODE:

$$\left[r + \pi + \lambda p_t\right] V\left(p_t\right) = Y\lambda p_t \left(\frac{r + \pi + \lambda}{r + \pi}\right) - \widetilde{w}\left(\frac{r + \pi + \lambda p_t}{r + \pi}\right) - V'\left(p_t\right) p_t \left(1 - p_t\right) \lambda \quad (1)$$

We can use this equation with the value matching condition ($V\left(p_{ss}^*\right) = 0$) and the smooth-pasting condition ($V'\left(p_{ss}^*\right) = 0$) to determine p_{ss}^* explicitly. We thus obtain

$$p_{ss}^* = \frac{\widetilde{w}\left(r + \pi\right)}{\lambda\left[Y\left(r + \lambda + \pi\right) - \widetilde{w}\right]}$$

Continuing toward solving the above ODE, a particular solution is the expected value of committing forever to the current employee:

$$\frac{\lambda p_t Y - \widetilde{w}}{r + \pi}$$

To capture the option value of being able to terminate the match, we must look to the solution of the homogeneous part of the ODE, which will have the form $\left(1 - p_t\right)^{1+\mu} p_t^{-\mu}$ for some μ to be determined. Applying techniques drawn from Bellman and Cooke (1963), Presman (1990), and Keller, Rady, and Cripps (2005), we obtain a solution of the form:

$$V\left(p_t\right) = \frac{\lambda p_t Y - \widetilde{w}}{r + \pi} + K\left(1 - p_t\right)^{\frac{\lambda + r + \pi}{\lambda}} p_t^{-\frac{r+\pi}{\lambda}} \quad (2)$$

Here, K is a constant to be determined by our boundary conditions. We obtain K in terms of the threshold p_{ss}^* by substituting equation (2) into equation (1) above at the belief $p_t = p_{ss}^*$. Obviously, value-matching and smooth-pasting must again be satisfied; using these, we obtain

$$K = \left(\frac{\widetilde{w} - \lambda p_{ss}^* Y}{r + \pi}\right)\left(\frac{1}{1 - p_{ss}^*}\right)\left[\left(\frac{p_{ss}^*}{1 - p_{ss}^*}\right)\right]^{\frac{r+\pi}{\lambda}}$$

From this, we can obtain the end result simply by substituting for $p_{ss}^* = \frac{\widetilde{w}(r+\pi)}{\lambda[Y(r+\lambda+\pi)-\widetilde{w}]}$ and simplifying. ■

---

[42] We have already established that $w = \widetilde{w}$ in this equilibrium.

Proposition 3: *In a nontrivial steady-state employment equilibrium:*

(i) *Without receiving a payoff Y, a firm will wait for time t_{ss}^* after hiring a worker before firing him, where*

$$t_{ss}^* = \frac{1}{\lambda} \ln \left[\left(\frac{1 - p_{ss}^*}{p_{ss}^*} \right) \left(\frac{q_{U(ss)}}{\alpha \left(1 - q_{U(ss)} \right)} \right) \right]$$

(ii) *Unemployed workers are hired at intensity:*

$$\eta_{ss} = \pi E_{ss} dt \left[\frac{q_{U(ss)} + \alpha \left(1 - q_{U(ss)} \right)}{q_{U(ss)} \left[1 - e^{-(\lambda + \pi) t_{ss}^*} \right] + \alpha \left(1 - q_{U(ss)} \right) \left[1 - e^{-\pi t_{ss}^*} \right]} \right]$$

(iii) *The employment level can be written:*

$$E_{ss} = \left[\frac{Q - q_{U(ss)}}{1 - q_{U(ss)}} \right] \left[1 + \left(\frac{\alpha}{q_{U(ss)}} \right) \left(\frac{1 - e^{-\pi t_{ss}^*}}{1 - e^{-(\pi + \lambda) t_{ss}^*} - \alpha \left[1 - e^{-\pi t_{ss}^*} \right]} \right) \right]$$

Proof (i): *If a firm receives no payoff at time t, the updating rules imply that $\frac{\partial (\ln(p_t))}{\partial t} = -\lambda \left(1 - p_t \right)$ and $\frac{\partial (\ln(1 - p_t))}{\partial t} = \lambda p_t$. Thus we can relate these two derivatives by*

$$\frac{\partial \left(\ln \left(1 - p_t \right) \right)}{\partial t} = \frac{\partial \left(\ln \left(p_t \right) \right)}{\partial t} + \lambda$$

Integrating both sides from 0 to t_{ss}^* yields

$$\ln \left(p_{t_{ss}^*} \right) - \ln \left(p_0 \right) + \lambda t_{ss}^* = \ln \left(1 - p_{t_{ss}^*} \right) - \ln \left(1 - p_0 \right)$$

where p_0 is the firm's initial belief about its employee's type. Note that (a) $p_{t_{ss}^*} = p_{ss}^*$, (b) p_0 must equal $q_H \left(q_{U(ss)} \right)$ in equilibrium, and (c) $\frac{q_H \left(q_{U(ss)} \right)}{1 - q_H \left(q_{U(ss)} \right)} = \frac{q_{U(ss)}}{\alpha \left(1 - q_{U(ss)} \right)}$, and the result follows. ∎

Proof (ii): *In equilibrium, the instantaneous flow into employment among the unemployed (η_{ss}) must equal the instantaneous flow out of employment among the employed. This flow out of employment at time t consists of both the measure of workers who quit jobs ($E_{ss} \pi dt$) and the measure of workers hired at time $t - t_{ss}^*$ whose employers received no payoff during that time:*

$$\eta_{ss} \left[\left[1 - q_H \left(q_{U(ss)} \right) \right] e^{-\pi t_{ss}^*} + q_H \left(q_{U(ss)} \right) e^{-(\lambda + \pi) t_{ss}^*} \right]$$

Equating these inflows and outflows yields

$$\eta_{ss} = \left[\frac{\pi E_{ss} dt}{1 - e^{-\pi t_{ss}^*} \left(1 - q_H \left(q_{U(ss)} \right) \right) - q_H \left(q_{U(ss)} \right) e^{-(\pi + \lambda) t_{ss}^*}} \right]$$

into which we can substitute $q_H \left(q_{U(ss)} \right) \equiv \frac{q_{U(ss)}}{\alpha \left(1 - q_{U(ss)} \right)}$ to obtain our desired result. ∎

Proof (iii): *In addition to the expression for η_{ss} obtained above (by equating inflows to and outflows from employment), we can also obtain η_{ss} by equating type H inflows to and type H outflows from employment (this must also hold in a steady-state employment equilibrium). From this, we obtain*

$$\eta_{ss} = \frac{E_{ss} q_{E(ss)} \pi dt}{q_H \left(q_{U(ss)} \right) \left[1 - e^{-(\lambda+\pi)t_{ss}^*} \right]} \quad where \quad q_{E(ss)} = \frac{Q - (1 - E_{ss}) q_{U(ss)}}{E_{ss}}$$

Equating the two expressions for η_{ss} and solving for E_{ss} yields the result. ∎

Proposition 4: *$\exists \bar{z} > 0$ such that for $z \in (0, \bar{z})$, the proportion of type H workers in the unemployment pool immediately following the shock ($Y \longrightarrow Y - z$) falls to*

$$q_{U(\hat{t})} = \frac{(1 - E_{ss}) q_{U(ss)} + \frac{\eta_{ss}}{dt} \displaystyle\int_{t_z}^{t_{ss}^*} q_H \left(q_{U(ss)} \right) e^{-(\pi+\lambda)s} ds}{1 - E_{ss} + \frac{\eta_{ss}}{dt} \displaystyle\int_{t_z}^{t_{ss}^*} \left[q_H \left(q_{U(ss)} \right) e^{-(\pi+\lambda)s} + \left(1 - q_H \left(q_{U(ss)} \right) \right) e^{-\pi s} \right] ds} < q_{U(ss)}$$

Proof: *This expression follows from the same intuition given for the mass of firings immediately following the shock (the numerator consists of only the type H workers from this mass).*

To show that the inequality holds for sufficiently small z, we can substitute for E_{ss}, η_{ss}, t_{ss}^*, t_z, p_{ss}^*, $p_{\hat{t}}^*$, and $q_H \left(q_{U(ss)} \right)$, and we can evaluate the expression. For small z, it is straightforward to see that this inequality will be satisfied if and only if $p_{ss}^* < q_{U(ss)}$. This condition is implied by our limit on firms' ex ante information (A1):

$$\alpha > \left(\frac{r + \pi}{r + \pi + \lambda} \right) \left(\frac{\widetilde{w}}{\lambda Y - \widetilde{w}} \right) \left(\frac{1 - q_H^*}{q_H^*} \right)$$

$$\implies \alpha > \left(\frac{r + \pi}{r + \pi + \lambda} \right) \left(\frac{\widetilde{w}}{\lambda Y - \widetilde{w}} \right) \left(\frac{\alpha \left(1 - q_{U(ss)} \right)}{q_{U(ss)}} \right)$$

$$\implies q_{U(ss)} (r + \pi + \lambda) (\lambda Y - \widetilde{w}) > (r + \pi) \widetilde{w} \left(1 - q_{U(ss)} \right)$$

$$\implies q_{U(ss)} > \frac{\widetilde{w} (r + \pi)}{\lambda \left[(r + \pi + \lambda) Y - \widetilde{w} \right]} = p_{ss}^*$$

so we have the result. ∎

Lemma 2: *$q_U^* (Y)$ is strictly decreasing in Y.*

Proof: *Clearly $p_{ss}^* = \frac{\widetilde{w}(r+\pi)}{\lambda[Y(r+\lambda+\pi)-\widetilde{w}]}$ is strictly decreasing in Y. Note that, for a given $p_t \in [p_{ss}^*, 1]$, $V(p_t)$ is strictly decreasing in the threshold p_{ss}^*. In turn, the $q_U^* (Y)$ satisfying $V (q_H (q_U^* (Y))) = c$ is strictly increasing in p_{ss}^* (and $q_H (q)$ is of course increasing in q). Thus, $q_U^* (Y)$ must be strictly decreasing in Y.* ∎

Proposition 5: *After the output shock $Y \longrightarrow Y - z$, hiring will cease for the duration $\widehat{t_H}$. If $q_U^* (Y - z) < Q$, $\widehat{t_H}$ is finite and satisfies*

$$q_U^* (Y - z) = f_1 \left(\widehat{t_H} \,\middle|\, t_{ss}^*, t_z \right)$$

$$\equiv \frac{
\begin{aligned}
& (1 - E_{ss}) \, q_{U(ss)} + \eta_{ss} \int_{t_z}^{t_{ss}^*} e^{-(\pi+\lambda)s} q_H \left(q_{U(ss)} \right) ds + \eta_{ss} e^{-(\pi+\lambda)t_z} q_H \left(q_{U(ss)} \right) \widehat{t_F} \\
& \quad + \eta_{ss} \int_0^{\widehat{t_F}} \pi e^{-\pi s} \int_s^{t_z} e^{-(\pi+\lambda)x} q_H \left(q_{U(ss)} \right) dx\,ds + \left[1 - e^{-\pi \widehat{t_H}} \right] m_{H(ss)} \\
& \quad + \eta_{ss} q_H \left(q_{U(ss)} \right) \int_0^{\widehat{t_H}} \left[e^{-\pi s} - e^{-\pi \widehat{t_H}} \right] \lambda e^{-\lambda s} \int_s^{t_z} e^{-(\pi+\lambda)x} dx\,ds
\end{aligned}
}{
\begin{aligned}
& (1 - E_{ss}) + \eta_{ss} \int_{t_z}^{t_{ss}^*} \left[e^{-(\pi+\lambda)s} q_H \left(q_{U(ss)} \right) + e^{-\pi s} \left(1 - q_H \left(q_{U(ss)} \right) \right) \right] ds \\
& \quad + \eta_{ss} \left[e^{-(\pi+\lambda)t_z} q_H \left(q_{U(ss)} \right) + e^{-\pi t_z} \left(1 - q_H \left(q_{U(ss)} \right) \right) \right] \widehat{t_F} \\
& \quad + \eta_{ss} \int_0^{\widehat{t_F}} \pi e^{-\pi s} \int_s^{t_z} \left[e^{-(\pi+\lambda)x} q_H \left(q_{U(ss)} \right) + e^{-\pi x} \left(1 - q_H \left(q_{U(ss)} \right) \right) \right] dx\,ds
\end{aligned}
}$$

$$+ \left[1 - e^{-\pi \widehat{t_H}} \right] m_{H(ss)} + \eta_{ss} q_H \left(q_{U(ss)} \right) \int_0^{\widehat{t_H}} \left[e^{-\pi s} - e^{-\pi \widehat{t_H}} \right] \lambda e^{-\lambda s} \int_s^{t_z} e^{-(\pi+\lambda)x} dx\,ds$$

where $m_{H(ss)} = \left(\frac{\lambda}{\pi} \right) \left(\frac{\eta_{ss}}{dt} \right) q_H \left(q_{U(ss)} \right) \int_0^{t_{ss}^*} e^{-(\pi+\lambda)s} ds$ is the steady-state mass of employed work-ers who have already revealed themselves to be type H and $\widehat{t_F} \equiv \min \left\{ t_z, \widehat{t_H} \right\}$ is the time after the shock during which previously hired workers were being fired at the new standard.

Proof: *First, note that if $q_U^* (Y - z) \geq Q$, $\widehat{t_H} = \infty$ (hiring can never resume with output at $Y - z$). As mentioned following Proposition 5 in the text, there can be no positive selection into unemployment, so $q_{U(t)} < Q$, $\forall t$. Clearly, then, the random inflows to unemployment from job quitters can–at most–bring the unemployment pool quality asymptotically toward Q. It can never reach any quality level $q_{U(t)} > Q$, and it can never reach quality level $q_{U(t)} = Q$ in finite time.*

To see that this pool will reach any $q_{U(t)} < Q$ in finite time, first recall that, after the shock, targeted firings continue to occur at the new belief threshold $p_{\widehat{t}}^*$ for those workers who were hired in the previous steady state (before the shock). Suppose that we have reached time $\widehat{t} + t_z$ (where t_z again represents the time after hiring after the shock when workers who have not yet revealed themselves to be type H will be fired) and that hiring has not yet begun. (Obviously, if hiring begins before this point, we have already reached $q_{U(t)} = q_U^* (Y - z)$, so we are done). Then the only remaining employed workers must have provided their employers a payoff, and these must be type H workers. Then of course, the type H proportion among the inflow to unemployment (which comes entirely through voluntary quits) must be 1. Over time, this flow to unemployment will raise the unemployment pool quality asymptotically toward Q, and by the structure of the Poisson distribution, it must surpass any $q < Q$ in finite time.

Regarding the expression pinning down $\widehat{t_H}$—note that this is simply a mathematical transla-

tion of the aggregate type H proportion among the six groups listed following Proposition 6 in the text. For clarity, the components are classified specifically according to these groups below:

$$\overbrace{(1-E_{ss})\,q_{U(ss)}}^{(i)} + \overbrace{\eta_{ss}\int_{t_z}^{t_{ss}^*} e^{-(\pi+\lambda)s}q_H\left(q_{U(ss)}\right)ds}^{(ii)} + \overbrace{\eta_{ss}e^{-(\pi+\lambda)t_z}q_H\left(q_{U(ss)}\right)\widehat{t_F}}^{(iii)}$$

$$+\overbrace{\eta_{ss}\int_0^{\widehat{t_F}}\pi e^{-\pi s}\int_s^{t_z}e^{-(\pi+\lambda)x}q_H\left(q_{U(ss)}\right)dxds}^{(iv)} + \overbrace{\left[1-e^{-\pi\widehat{t_H}}\right]m_{H(ss)}}^{(v)}$$

$$+\overbrace{\eta_{ss}q_H\left(q_{U(ss)}\right)\int_0^{\widehat{t_H}}\left[e^{-\pi s}-e^{-\pi\widehat{t_H}}\right]\lambda e^{-\lambda s}\int_s^{t_z}e^{-(\pi+\lambda)x}dxds}^{(vi)}$$

$$\overbrace{1-E_{ss}}^{(i)} + \overbrace{\eta_{ss}\int_{t_z}^{t_{ss}^*}\left[e^{-(\pi+\lambda)s}q_H\left(q_{U(ss)}\right)+e^{-\pi s}\left(1-q_H\left(q_{U(ss)}\right)\right)\right]ds}^{(ii)}$$

$$+\overbrace{\eta_{ss}\left[e^{-(\pi+\lambda)t_z}q_H\left(q_{U(ss)}\right)+e^{-\pi t_z}\left(1-q_H\left(q_{U(ss)}\right)\right)\right]\widehat{t_F}}^{(iii)}$$

$$+\overbrace{\eta_{ss}\int_0^{\widehat{t_F}}\pi e^{-\pi s}\int_s^{t_z}\left[e^{-(\pi+\lambda)x}q_H\left(q_{U(ss)}\right)+e^{-\pi x}\left(1-q_H\left(q_{U(ss)}\right)\right)\right]dxds}^{(iv)}$$

$$+\overbrace{\left[1-e^{-\pi\widehat{t_H}}\right]m_{H(ss)}}^{(v)} + \overbrace{\eta_{ss}q_H\left(q_{U(ss)}\right)\int_0^{\widehat{t_H}}\left[e^{-\pi s}-e^{-\pi\widehat{t_H}}\right]\lambda e^{-\lambda s}\int_s^{t_z}e^{-(\pi+\lambda)x}dxds}^{(vi)}$$

Note in (iii) and (iv) that, if all remaining unproductive workers hired at the previous steady-state have been fired before hiring restarts, then $\widehat{t_F}=t_z$. If hiring begins while unproductive workers are still reaching this threshold, then $\widehat{t_F}=\widehat{t_H}$. ∎

Proposition 6: *Consider an unanticipated transitory output shock $Y\longrightarrow Y-z$ to the steady-state at time $\widehat{t}$, immediately after which it is known that output will rebound $Y-z\longrightarrow Y$ at Poisson-distributed times with parameter γ. After firms respond optimally to this shock at $\widehat{t}$, if the recovery occurs before time $\widehat{t}+\widehat{t_{Y-z,Y}}$ (where $\widehat{t_{Y-z,Y}}$ satisfies $q_U^*\left(Y\right)=f_2\left(\widehat{t_{Y-z,Y}}\,\middle|\,t_{ss}^*,t_{Y-z}^*\right)$, and an expression for $f_2\left(\widehat{t_{Y-z,Y}}\,\middle|\,t_{ss}^*,t_{Y-z}^*\right)$ is provided in the appendix),* **then the economy will remain without hiring for a positive duration of time even after the recovery.** *Further, $\exists\overline{z}>0$ such that for $z\in(0,\overline{z})$, both the likelihood and expected duration of a jobless recovery are increasing in the magnitude of the shock z.*

Proof: $f_2\left(\widehat{t_{Y-z,Y}}\,\Big|\,t_{ss}^*, t_{Y-z}^*\right)$ must express the proportion of type H workers in the unemployment pool at time $\widehat{t} + \widehat{t_{Y-z,Y}}$ (after the initial shock but before the recovery). The intuition for constructing this expression will involve the same six groups used to construct $f_1\left(\widehat{t_H}\,\Big|\,t_{ss}^*, t_z\right)$ in Proposition 6. In fact, given the same arguments $\widehat{t_{Y-z,Y}}$, t_{ss}^*, and t_{Y-z}^*, the function $f_2\left(\widehat{t_{Y-z,Y}}\,\Big|\,t_{ss}^*, t_{Y-z}^*\right)$ is identical to $f_1\left(\widehat{t_{Y-z,Y}}\,\Big|\,t_{ss}^*, t_{Y-z}^*\right)$. To understand this, notice that a recovery occurring before $\widehat{t} + \widehat{t_{Y-z,Y}}$ will decrease both the firing threshold p^* and the hiring threshold q^* (from $q_U^*\,(Y-z)$ to $q_U^*\,(Y)$). The drop in p^* will result in delayed firings, but no immediate firm response. In turn, if hiring would not begin at the new, lower $q^* = q_U^*\,(Y)$, this drop in q^* would cause no immediate firm response either. Thus, for a recovery at a time $t \in \left[\widehat{t}, \widehat{t} + \widehat{t_{Y-z,Y}}\right)$, the unemployment pool and its quality will not change discretely in response to the recovery. Given this, we know that $\widehat{t_{Y-z,Y}}$ must satisfy:

$$
q_U^*\,(Y) = f_2\left(\widehat{t_{Y-z,Y}}\,\Big|\,t_{ss}^*, t_{Y-z}^*\right)
$$

$$
\equiv \frac{
\begin{aligned}
&(1 - E_{ss})\, q_{U(ss)} + \eta_{ss} \int_{t_{Y-z,Y}^*}^{t_{ss}^*} e^{-(\pi+\lambda)s} q_H\left(q_{U(ss)}\right) ds + \eta_{ss} e^{-(\pi+\lambda)t_{Y-z,Y}^*} q_H\left(q_{U(ss)}\right)\widehat{t_F}\\
&+\eta_{ss}\int_0^{\widehat{t_F}} \pi e^{-\pi s} \int_s^{t_{Y-z,Y}^*} e^{-(\pi+\lambda)x} q_H\left(q_{U(ss)}\right) dx\, ds + \left[1 - e^{-\pi\widehat{t_{Y-z,Y}}}\right] m_{H(ss)}\\
&+\eta_{ss} q_H\left(q_{U(ss)}\right)\int_0^{\widehat{t_{Y-z,Y}}}\left[e^{-\pi s} - e^{-\pi\widehat{t_{Y-z,Y}}}\right]\lambda e^{-\lambda s}\int_s^{t_{Y-z,Y}^*} e^{-(\pi+\lambda)x} dx\, ds
\end{aligned}
}{
\begin{aligned}
&(1 - E_{ss}) + \eta_{ss}\int_{t_{Y-z,Y}^*}^{t_{ss}^*}\left[e^{-(\pi+\lambda)s} q_H\left(q_{U(ss)}\right) + e^{-\pi s}\left(1 - q_H\left(q_{U(ss)}\right)\right)\right] ds\\
&+\eta_{ss}\left[e^{-(\pi+\lambda)t_{Y-z,Y}^*} q_H\left(q_{U(ss)}\right) + e^{-\pi t_{Y-z,Y}^*}\left(1 - q_H\left(q_{U(ss)}\right)\right)\right]\widehat{t_F}\\
&+\eta_{ss}\int_0^{\widehat{t_F}} \pi e^{-\pi s}\int_s^{t_{Y-z,Y}^*}\left[e^{-(\pi+\lambda)x} q_H\left(q_{U(ss)}\right) + e^{-\pi x}\left(1 - q_H\left(q_{U(ss)}\right)\right)\right] dx\, ds\\
&+\left[1 - e^{-\pi\widehat{t_{Y-z,Y}}}\right] m_{H(ss)} + \eta_{ss} q_H\left(q_{U(ss)}\right)\int_0^{\widehat{t_{Y-z,Y}}}\left[e^{-\pi s} - e^{-\pi\widehat{t_{Y-z,Y}}}\right]\lambda e^{-\lambda s}\int_s^{t_{Y-z,Y}^*} e^{-(\pi+\lambda)x} dx\, ds
\end{aligned}
}
$$

where $m_{H(ss)} = \left(\frac{\lambda}{\pi}\right)\left(\frac{\eta_{ss}}{dt}\right) q_H\left(q_{U(ss)}\right)\int_0^{t_{ss}^*} e^{-(\pi+\lambda)s} ds$ is the steady-state mass of employed workers who have already revealed themselves to be type H. As in previous results, $\widehat{t_F} \equiv \min\left\{t_z, \widehat{t_{Y-z,Y}}\right\}$.

If this $f_2\left(\widehat{t_{Y-z,Y}}\,\Big|\,t_{ss}^*, t_{Y-z}^*\right)$ has not risen above $q_U^*\,(Y)$ by the time the recovery occurs, this recovery will have no discrete effect on the unemployment pool quality. Thus, in this case, there will be a period without hiring even after the recovery.

Note that the distribution of recovery times depends only on γ. Since $\widehat{t_{Y-z,Y}} = 0$ at $z = 0$ and $\widehat{t_{Y-z,Y}} > 0$ for positive z, a larger value of $\widehat{t_{Y-z,Y}}$ implies a greater likelihood of a jobless recovery (note also that this moves continuously in z). Since the expected duration of a jobless recovery was 0 at $z = 0$, this is increasing in $\widehat{t_{Y-z,Y}}$ as well. Thus, we can show that a jobless recovery's

likelihood and expected duration are increasing in z by establishing that $\widehat{t_{Y-z,Y}}$ is increasing in z.

To see that $\widehat{t_{Y-z,Y}}$ is increasing in z for a range of z, notice first that the post-shock unemployment pool quality $q_{U(\hat{t})}$ is decreasing in z for z sufficiently small (since we consider only equilibria for which $p_{ss}^* < q_{U(ss)}$). Increases in z on this range therefore lead to lower $q_{U(\hat{t})}$, but we must consider also that the post-shock flow to unemployment will come from a combination of random job-quitters and workers fired at the higher threshold $p^*(Y-z,Y)$. Because of this, we might worry that a faster rate of recovery might overcome the lower starting quality. Of course, this higher threshold will also yield a more intense flow of directed firings into the unemployment pool after the shock, and this will mitigate improvement in the rate of recovery caused by the higher threshold.

In line with this reasoning, it turns out that the post-shock time required to reach the unemployment quality threshold $q_U^*(Y)$ is increasing in z for small z, which is our desired result. This can be verified by differentiating $f_2\left(\widehat{t_{Y-z,Y}} \middle| t_{ss}^*, t_{Y-z}^*\right)$ with respect to z at the point $z=0$. I omit an expression for this derivative because it is algebraically complex and economically uninsightful, even though this result has straightforward intuition. ∎

In addition to the above result, I will provide an alternate formulation of this in which the initial unanticipated output shock $Y \longrightarrow Y-z$ at time $\hat{t}$ is followed almost immediately by another unanticipated output shock $Y-z \longrightarrow Y$ at time $\hat{t}+dt$. (Firms have already responded to the first shock before the second occurs.) Of course, a setting with two unanticipated shocks that are both expected to be permanent is farther from reality than the environment of Proposition 6, but the result is similar, and the simplicity of this setting allows us to connect the result to clear intuition.

Proposition 6.A: After an unanticipated output shock $Y \longrightarrow Y-z$ at time $\hat{t}$, firm responses to this output shock, and an unanticipated perfect reversal of this output shock $Y-z \longrightarrow Y$ at time $\hat{t}+dt$, hiring will cease for duration $\widehat{\widehat{t_H}} > 0$.[43]

Proof: First, we can bound $\widehat{\widehat{t_H}}$ above by $t_{ss}^* - t_z$. To see this, note that the mass of firings in response to the shock (in the belief range $\left[p_{ss}^*, p_{\hat{t}}^*\right)$) will be of better quality than the directed firings that would have occurred at belief p_{ss}^* without the shock. This is because some of those fired in response to the shock would have revealed themselves to be type H workers during the subsequent period of length $t_{ss}^* - t_z$, but these type H workers are instead included in

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[43]Note that $\widehat{\widehat{t_H}}$ satisfies the condition $q_U^*(Y) = f_2\left(\widehat{\widehat{t_H}} \middle| t_{ss}^*, t_z\right)$, where $f_2\left(\widehat{\widehat{t_H}} \middle| t_{ss}^*, t_z\right)$ is analogous to the expression $f_1\left(\widehat{t_H} \middle| t_{ss}^*, t_z\right)$ in Proposition 6, but with terms adjusted to account for the firing and hiring thresholds immediately returning to their previous levels.

the mass firings after the shock. Additionally, until hiring begins, this downward pressure on the unemployment pool quality (which would have been present without the shock) will be absent. These two facts imply that, after the elapsed time $t_{ss}^* - t_z$, without hiring beginning, the unemployment pool quality without hiring beginning must be strictly higher after the two-shock even than it would have been after no shock at all. The unemployment pool quality threshold for hiring $(q_U^*(Y))$ is identical in both cases (since Y returns to the same level after the second shock), and the unemployment pool quality without any shock will remain at precisely $q_U^*(Y)$. Thus, after the elapsed time period $t_{ss}^* - t_z$ following the two-shock sequence, the unemployment pool quality would be greater than $q_U^*(Y)$ without hiring beginning, so hiring must begin again before this time $t_{ss}^* - t_z$ has elapsed.

Given this bound, we can provide an analogous expression to that from Proposition 6 for the time at which hiring will be renewed (note that group (iii) is no longer included, since the threshold returns to p_{ss}^*, and since hiring must begin again before any workers who aren't fired in the initial response to the shock reach this threshold):

$$q_U^*(Y) = f_2\left(\widehat{\widehat{t_H}}\,\middle|\,t_{ss}^*, t_z\right)$$

$$= \frac{
\overbrace{(1 - E_{ss})\,q_{U(ss)}}^{(i)} + \overbrace{\eta_{ss}\int_{t_z}^{t_{ss}^*} e^{-(\pi+\lambda)s} q_H\left(q_{U(ss)}\right) ds}^{(ii)}
}{
\underbrace{}
}
$$

$$+ \overbrace{\eta_{ss}\int_0^{\widehat{t_H}} \pi e^{-\pi s}\int_s^{t_z+s} e^{-(\pi+\lambda)x} q_H\left(q_{U(ss)}\right) dx\,ds}^{(iv)} + \overbrace{\left[1 - e^{-\pi\widehat{t_H}}\right] m_{H(ss)}}^{(v)}$$

$$+ \overbrace{\eta_{ss} q_H\left(q_{U(ss)}\right)\int_0^{\widehat{t_H}} \left[e^{-\pi s} - e^{-\pi\widehat{t_H}}\right]\lambda e^{-\lambda s}\int_s^{t_z+s} e^{-(\pi+\lambda)x} dx\,ds}^{(vi)}$$

$$= \frac{\cdots}{
\overbrace{1 - E_{ss}}^{(i)} + \overbrace{\eta_{ss}\int_{t_z}^{t_{ss}^*}\left[e^{-(\pi+\lambda)s} q_H\left(q_{U(ss)}\right) + e^{-\pi s}\left(1 - q_H\left(q_{U(ss)}\right)\right)\right] ds}^{(ii)}
} \quad\blacksquare$$

$$+ \overbrace{\eta_{ss}\int_0^{\widehat{t_H}} \pi e^{-\pi s}\int_s^{t_z+s}\left[e^{-(\pi+\lambda)x} q_H\left(q_{U(ss)}\right) + e^{-\pi x}\left(1 - q_H\left(q_{U(ss)}\right)\right)\right] dx\,ds}^{(iv)}$$

$$+ \overbrace{\left[1 - e^{-\pi\widehat{t_H}}\right] m_{H(ss)}}^{(v)} + \overbrace{\eta_{ss} q_H\left(q_{U(ss)}\right)\int_0^{\widehat{t_H}}\left[e^{-\pi s} - e^{-\pi\widehat{t_H}}\right]\lambda e^{-\lambda s}\int_s^{t_z+s} e^{-(\pi+\lambda)x} dx\,ds}^{(vi)}$$

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APPENDIX B: Extensions

Mechanics of Equilibrium with Varying Employment (From Section 6.2)

Here I discuss in greater detail the equilibria mentioned in Section 6.2. Again, the free entry condition requires the quality of the unemployment pool to be constant. In order to maintain this constant quality, the combined negative pressure from selective hirings and targeted firings must precisely counter the positive pressure from quitting workers. In equilibrium, the economy preserves this balance by adjusting the intensity of hiring to account for the disparity between the positive pressure of quitting workers and the negative pressure of fired workers. For purposes of intuition, it is worth remembering that these conditions regarding the intensity of hiring reflect optimal firm behavior—firms will continue to hire as long as it is profitable to do so.

Of course, it is crucial to note that, in this model, the intensity of targeted firings at t is determined by the intensity of selective hirings at $t - t_v^*$. Hence, the intensity of hiring necessary to preserve equilibrium at time t is a function of the current employment level E_t and the hiring intensity $t - t_v^*$ earlier $(\eta_{t-t_v^*})$.[44]

Let us formalize this by defining $\eta_v \left(\eta_{t-t_v^*}, E_t \right) : \mathbb{R}_+ \times [0,1] \longrightarrow \mathbb{R}$. To preserve the type H proportion of the unemployment pool, the hiring intensity must be that at which the net flow into/out of unemployment will have proportion $q_{U(v)}$ of type H workers—this is the same type H proportion as the unemployment pool itself. Thus, $\eta_v \left(\eta_{t-t_v^*}, E_t \right)$ must satisfy:

$$
q_{U(v)} = \frac{\overbrace{\eta_{t-t_v^*} q_H \left(q_{U(v)} \right) e^{-(\lambda+\pi)t_v^*}}^{\text{directed firings}} + \overbrace{E_t q_{E(t)} \pi dt}^{\text{random quits}} - \overbrace{\eta_v \left(\eta_{t-t_v^*}, E_t \right) q_H \left(q_{U(v)} \right)}^{\text{selective hirings}}}{\underbrace{\eta_{t-t_v^*} \left[\left[1 - q_H \left(q_{U(v)} \right) \right] e^{-\pi t_v^*} + q_H \left(q_{U(v)} \right) e^{-(\lambda+\pi)t_v^*} \right]}_{\text{directed firings}} + \underbrace{E_t \pi dt}_{\text{random quits}} - \underbrace{\eta_v \left(\eta_{t-t_v^*}, E_t \right)}_{\text{selective hirings}}}
$$

Obviously, there cannot be a negative intensity of hiring, so if $\eta_v \left(\eta_{t-t_v^*}, E_t \right) < 0$, the unemployment pool quality will fall below $q_{U(v)}$ and hiring will stop for some amount of time until the pool quality again rises to $q_{U(v)}$. This can occur if screening has limited effectiveness (α is not too close to 0)—so much hiring takes place at some point in time that conditions will require hiring to stop $t - t_v^*$ later.[45]

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[44]Note that we still must satisfy the accounting condition $E_t q_{U(v)} + (1 - E_t) q_{E(t)} = Q$, and that all equilibria we are considering involve the same unemployment pool quality level $q_{U(v)}$. Hence, a given employment level E_t necessarily implies a unique type H proportion among the employed $q_{E(t)}$. (Of course, if the level E_t implies a $q_{E(t)} \notin [0,1]$, such an E_t level cannot be consistent with equilibrium. Further, an economic state with an implied $q_{E(t)} \in [0, Q)$ could not be reached by any of the forces considered in this paper.)

[45]In the case where screening is extremely precise (α close to 0), it is possible that $p_{ss}^* > q_{U(ss)}$. As a result, $\eta_v \left(\eta_{t-t_v^*}, E_t \right) > 0$ and free entry always holds with equality.

Example: Cyclical Equilibrium

In the general description above, we required no repeated cyclical employment pattern—employment evolved simply to preserve the free entry condition. For certain parameter combinations, though, it is possible to construct an equilibrium in which there are self-sustaining cycles in hiring and firing. To see this, consider a simple example:

Imagine an equilibrium with 2 intensities of hiring, η_H and η_L (assume $\eta_H > \eta_L$). This equilibrium will consist of repeated cycles with an expansionary period of length t_v^* during which $\eta_t = \eta_H$, followed by a contractionary period of length t_v^* with $\eta_t = \eta_L$. Obviously, the employment level grows during the expansionary period and shrinks during the contractionary period.[46] To preserve the cyclicality, the total growth while $\eta_t = \eta_H$ must equal the total contraction while $\eta_t = \eta_L$.

To understand why such an equilibrium can preserve the unemployment pool quality level, notice that whenever $\eta_t = \eta_H$, it must also be true that $\eta_{t-t_v^*} = \eta_L$. Similarly, these intensities are reversed during contractions. Hence, the hiring and firing intensities during an expansionary period must support a net flow into unemployment with type H proportion $q_{U(v)}$, and these intensities during a contractionary period must support a net flow out of unemployment with type H proportion $q_{U(v)}$. In the equation satisfied by $\eta_v\left(\eta_{t-t_v^*}, E_t\right)$ given above, the numerator and denominator are both positive during expansionary periods and both negative during contractionary periods.

Let us now consider the aggregate conditions that must be satisfied to sustain these cycles (taking optimal firm behavior and the corresponding thresholds p_v^* and $q_{U(v)} = q_U^*(Y)$ as given). For ease of notation, define:

- the net change in the employment level from t to $t + dt$:

$$\Delta_E(t) \equiv E_{t+dt} - E_t$$

- the mass of type H workers employed at time t:

$$E_{H(t)} \equiv q_{E(t)} E_t$$

- the net change in the mass of type H workers employed from t to $t + dt$:

$$\begin{aligned} \Delta_{E(H)}(t) &\equiv q_{E(t+dt)} E_{t+dt} - q_{E(t)} E_t \\ &= E_{H(t+dt)} - E_{H(t)} \end{aligned}$$

---

[46] In turn, $q_{E(t)}$ falls during expansions and rises during contractions. So, in line with standard intuition, the average quality of employed workers is greatest when employment is lowest—toward the end of a contractionary period.

Further, index times in the cycle by $t \in [0, 2t_v^*]$, where $t \in [0, t_v^*)$ correspond to the expansionary part of the cycle, and $t \in [t_v^*, 2t_v^*)$ correspond to the contractionary part. With this notation established, this cyclical employment equilibrium must satisfy the following:

The labor force must always be of unit mass, and it must have the proportion Q of type H workers, so the employment level and type H proportion of those employed must reflect this at all times:

$$E_{H(t)} + (1 - E_t) q_{U(v)} = Q \text{ for } t \in [0, 2t_v^*]$$

The employment level and the mass of type H workers employed must evolve according to the net flows into each. This net flow should consist of hirings minus directed firings and quits. Thus

$$\Delta_E(t) = -E_t \pi dt + \eta_H - \eta_L \left[\left(1 - q_H \left(q_{U(v)} \right) \right) e^{-\pi t_v^*} + q_H \left(q_{U(v)} \right) e^{-(\lambda+\pi)t_v^*} \right] \text{ for } t \in [0, t_v^*]$$

$$\Delta_{E(H)}(t) = -\pi E_{H(t)} dt + q_H \left(q_{U(v)} \right) \left[\eta_H - \eta_L e^{-(\lambda+\pi)t_v^*} \right] \text{ for } t \in [0, t_v^*]$$

$$\Delta_E(t) = -E_t \pi dt + \eta_L - \eta_H \left[\left(1 - q_H \left(q_{U(v)} \right) \right) e^{-\pi t_v^*} + q_H \left(q_{U(v)} \right) e^{-(\lambda+\pi)t_v^*} \right] \text{ for } t \in [t_v^*, 2t_v^*]$$

$$\Delta_{E(H)}(t) = -\pi E_{H(t)} dt + q_H \left(q_{U(v)} \right) \left[\eta_L - \eta_H e^{-(\lambda+\pi)t_v^*} \right] \text{ for } t \in [t_v^*, 2t_v^*]$$

Further, as explained above, the net flow into employment must always have proportion $q_{U(v)}$ of type H workers (regardless of whether this net flow is positive or negative):

$$q_{U(v)} = \frac{\Delta_E(t)}{\Delta_{E(H)}(t)} \text{ for } t \in [0, 2t_v^*]$$

Finally, in order for the equilibrium to be truly cyclical, the net inflows to employment during the expansionary period must be exactly reversed by the net outflows from employment during the contractionary period:

$$\int_0^{t_v^*} \Delta_E(s) \, ds + \int_{t_v^*}^{2t_v^*} \Delta_E(s) \, ds = 0$$

Thus, for this "expansion/contraction" equilibrium to exist, the expansion/contraction periods must each last as long as any individual firm would wait without output before firing a worker, and the total growth of employment during this expansion must erode completely during the following contraction. Further, this must all occur with net flows to/from employment always having the same proportion of type H workers as the unemployment pool itself $(q_{U(v)})$.

APPENDIX C: Data, Empirical Methodology and Supplemental Results

CPS Monthly Employment Data

The Current Population Survey (CPS) data used are a monthly survey administered at the household-level. In addition to individual and household characteristics, these data also contain information about labor market participation and outcomes. Of particular use to the current study, individual observations in the data can be linked across months. This linking allows us to observe whether unemployed workers in a given month found employment by the following month, and these hiring outcomes serve as a dependent variable in my empirical analysis.

Each housing unit in the survey is interviewed for four consecutive months, dropped from the sample for eight months, and then brought back for another four months. This sampling structure is evenly distributed across months and years. In each month of data, it is year 1 in the sample for half of households and year 2 in the sample for the other half; further, $\frac{1}{8}$ of households are completely new to the sample, and $\frac{3}{4}$ of households will be included in the following month's sample as well. The success rate in linking individuals across months was quite high—among the $\frac{3}{4}$ of the sample expected to appear in the following sample, 94% were actually matched.

Note that this empirical approach does not require a longitudinal data structure—for this month's unemployed, I need only to observe the following month's employment status. The section below explains in more detail how these linked monthly data are used in the analysis.

Details of Results in Table 1 and Table 2

To obtain the values $\frac{\partial \ln(H_t^S)}{\partial \ln(Q_{t-1})} - \frac{\partial \ln(H_t^L)}{\partial \ln(Q_{t-1})}$ and $\frac{\partial \ln(H_t^S)}{\partial \ln(F_{t-1})} - \frac{\partial \ln(H_t^L)}{\partial \ln(F_{t-1})}$ reported in Tables 1 and 2, I estimate equations of the form

$$
\begin{aligned}
\partial \ln\left(H_t^i\right) &= \beta_0 + \beta_1 \partial \ln\left(H_t\right) + \beta_2 \partial \ln\left(F_{t-1}\right) + \beta_{2S} \mathbb{I}\left[i \in S\right] \partial \ln\left(F_{t-1}\right) \\
&\quad + \beta_3 \partial \ln\left(Q_{t-1}\right) + \beta_{3S} \mathbb{I}\left[i \in S\right] \partial \ln\left(Q_{t-1}\right) + \varepsilon_{it}
\end{aligned}
$$

where $\mathbb{I}\left[i \in S\right]$ is an indicator for whether individual i is among the short-term unemployed. Clearly, our estimate for $\widehat{\beta_{3S}}$ corresponds to $\frac{\partial \ln(H_t^S)}{\partial \ln(Q_{t-1})} - \frac{\partial \ln(H_t^L)}{\partial \ln(Q_{t-1})}$, while $\widehat{\beta_{2S}}$ corresponds to $\frac{\partial \ln(H_t^S)}{\partial \ln(F_{t-1})} - \frac{\partial \ln(H_t^L)}{\partial \ln(F_{t-1})}$.

Recall that H_t is the aggregate probability that time t unemployed workers are hired by $t+1$:

$$
H_t \equiv \frac{\text{total \# of unemployed hired at time } t}{\text{total \# of unemployed at time } t}
$$

and note that we use the discrete-time approximations $\partial \ln\left(H_t^i\right) \approx \frac{H_t^i - H_{t-1}^i}{H_{t-1}^i}$, $\partial \ln\left(H_t\right) \approx \frac{H_t - H_{t-1}}{H_{t-1}}$, $\partial \ln\left(F_t\right) \approx \frac{F_t - F_{t-1}}{F_{t-1}}$, and $\partial \ln\left(Q_t\right) \approx \frac{Q_t - Q_{t-1}}{Q_{t-1}}$. Our dependent variable $\partial \ln\left(H_t^i\right)$ is the only source

of individual-level variation in this estimation (beyond short/long-term unemployment status). We exploit individual-specific covariates in the data through this term, which we obtain in the following way:

Let x_i denote the k-dimensional vector of covariates specific to individual i, and note that these covariates are fixed over time for each individual. Using these covariates, we estimate equations of the form $y_{it} = g_t(x_i) + \varepsilon_{it}$ for each t, where y_{it} is a binary indicator for whether or not individual i (who was unemployed in period t) became employed in period $t+1$.[47] From this, we obtain the optimal functions $\widehat{g_t(\cdot)}$ for each period t, and H_t^i is simply $\widehat{g_t(x_i)}$. We thus approximate our dependent variable with

$$\partial \ln \left(H_t^i \right) \approx \frac{\widehat{g_t(x_i)} - \widehat{g_{t-1}(x_i)}}{\widehat{g_{t-1}(x_i)}}$$

Intuitively, $\widehat{g_t(x_i)}$ represents the time t reemployment probability of a worker with observables x_i. In turn $\widehat{g_t(x_i)} - \widehat{g_{t-1}(x_i)}$ is the change in reemployment probability (from time $t-1$ to time t) of a worker with observables x_i, and $\partial \ln \left(H_t^i \right)$ is the corresponding percent change. If $\widehat{g_t(x_i)} > \widehat{g_{t-1}(x_i)}$, then a worker with observables x_i is more likely to be hired in period t than in period $t-1$.

Suppose that firm hiring decisions are based on individual characteristics that correlate positively with quality and that at least some of these characteristics are not contained in x_i (meaning that they are unobservable to the econometrician). Then conditioning on x_i, a worker's probability of being hired at time t should correlate with her quality. In other words, firm hiring decisions in period t will inform us about the average quality of individuals with a given covariate vector x_i at time t. Thus, $\partial \ln \left(H_t^i \right) > 0$ indicates that the average quality of individuals with covariates x_i improved from $t-1$ to t; our dependent variable reflects unobserved worker quality. We can then extend this to the pools of short- and long-term unemployed to assess how unobserved worker quality changes for these two groups.

Additionally, it is worth recognizing the distinctions between specifications $I - IV$ in Tables 1 and 2. In Table 1, the estimates for specification I were obtained according to the process described in Section 2.2 and above. Individual-level observables used to compute $\partial \ln \left(H_t^i \right)$ were age, race, and unemployment duration (the results are similar if reasons for unemployment are also included). CPS-provided household sampling weights are used both in obtaining $\partial \ln \left(H_t^i \right)$ and in the regression to measure the effects of changes in firings/quits on $\partial \ln \left(H_t^i \right)$. Specification II differs only in that the latter regression does not control for aggregagate changes in hiring probabilities. In turn, specification III deviates from I only in that sampling weights are not used in the estimation. Finally, specification IV further restricts the observables used

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[47]For transparency, the analysis here imposes linearity on $g_t(\cdot)$, so we can write $\widehat{g_t(x_i)} \equiv \widehat{\beta} x_i$.

in obtaining $\partial \ln \left(H_t^i \right)$—in this case, x_i consists only of individual i's unemployment duration group, so this directly compares changes in the overall quality of workers in short- and long-term unemployment..

In Table 2, all estimates provided have been obtained via the same process as those in specification I of Table 1. However, specifications I and III include data from January 2001 - August 2011, while specifications II and IV include only the September 2008 - August 2011 window. Additionally, specifications III and IV include standard errors that have been clustered within each year-month pair; all standard errors reported in Table 1 use this clustering. The standard errors reported in specifications I and II, however, are not clustered.

The main results in these tables appear quite robust to these variations.

Table 2: Responses of STU - LTU hiring probabilities to firings/quits, including flows during recession (Source: CPS)

	(I)	(II)	(III)	(IV)
$\dfrac{\partial \ln\left(H_t^S\right)}{\partial \ln\left(F_{t-1}\right)} - \dfrac{\partial \ln\left(H_t^L\right)}{\partial \ln\left(F_{t-1}\right)}$	-0.101 (0.038)	-0.471 (0.184)	-0.101 (0.185)	-0.471 (0.460)
$\dfrac{\partial \ln\left(H_t^S\right)}{\partial \ln\left(Q_{t-1}\right)} - \dfrac{\partial \ln\left(H_t^L\right)}{\partial \ln\left(Q_{t-1}\right)}$	0.811 (0.465)	1.242 (0.746)	0.811 (0.792)	1.242 (1.305)
Standard errors clustered at year-month level?	N	N	Y	Y
Control for $\partial \ln\left(H_t\right)$?	Y	Y	Y	Y
Use sampling weights?	Y	Y	Y	Y
Condition on individual observables?	Y	Y	Y	Y
Time Period	Jan 2001 - Aug 2011	Sept 2008 - Aug 2011	Jan 2001 - Aug 2011	Sept 2008 - Aug 2011
N	87,518	33,459	87,518	33,459

CPS data on unemployment flows are used to obtain estimates

Sample used is restricted to unemployed men with no education beyond a HS diploma

Table 3: Summary Statistics for Unemployed Workers (Source: CPS)

Time Period:		Jan 2001 - Aug 2011		Jan 2001 - Aug 2008		Sept 2008 - Aug 2011	
		N	% Employed Next Month	N	% Employed Next Month	N	% Employed Next Month
All Reasons for Unemployment	All	380,519	43.0%	248,823	56.5%	131,696	17.7%
	Male	205,940	42.7%	130,335	56.6%	75,605	18.6%
	Male < HS Diploma	126,093	42.3%	80,120	56.0%	45,973	18.5%
Job Leavers / Losers only	All	250,984	44.0%	160,204	58.0%	90,780	19.2%
	Male	147,995	43.9%	91,792	58.3%	56,203	20.2%
	Male < HS Diploma	87,518	43.8%	54,059	58.1%	33,459	20.7%
Job Leavers / Losers only (Duration $\leq$ 12 weeks)	All	141,216	50.3%	102,270	58.7%	38,946	28.3%
	Male	81,967	50.3%	57,851	59.1%	24,116	29.1%
	Male < HS Diploma	49,849	50.2%	35,116	58.9%	14,733	29.6%
Job Leavers / Losers only (Duration > 12 weeks)	All	107,123	35.1%	55,289	56.4%	51,834	12.4%
	Male	64,308	35.1%	32,221	56.7%	32,087	13.5%
	Male < HS Diploma	36,641	34.6%	17,915	56.4%	18,726	13.7%